### Mathematics overview: Stage 10

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<th>Hours</th>
<th>Mastery indicators</th>
<th>Essential knowledge</th>
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<td>Investigating properties of shapes</td>
<td>12</td>
<td>• Manipulate fractional and negative indices</td>
<td>• Know the convention for labelling the sides in a right-angle triangle</td>
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<tr>
<td>Calculating</td>
<td>8</td>
<td>• Solve problems involving direct and inverse proportion</td>
<td>• Know the trigonometric ratios, sinθ = opposite/hypotenuse, cosθ = adjacent/hypotenuse, tanθ = opposite/adjacent</td>
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<tr>
<td>Solving equations and inequalities I</td>
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<td>• Convert between recurring decimals and fractions</td>
<td>• Know the exact values of sinθ and cosθ for θ = 0°, 30°, 45°, 60° and 90°</td>
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<td>Mathematical movement I</td>
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<td>• Know the exact value of tanθ for θ = 0°, 30°, 45° and 60°</td>
</tr>
<tr>
<td>Algebraic proficiency: tinkering</td>
<td>12</td>
<td>• Manipulate algebraic expressions by factorising a quadratic expression of the form $ax^2 + bx + c$</td>
<td>• Know that $a^{1/n} = ...$</td>
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<tr>
<td>Proportional reasoning</td>
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<td>• Solve quadratic equations by factorising</td>
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<td>Solving equations and inequalities II</td>
<td>6</td>
<td>• Interpret a gradient as a rate of change</td>
<td>• Know the special case of the difference of two squares</td>
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<td>Calculating space</td>
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<td>• Know how to set up an equation involving direct or inverse proportion</td>
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<tr>
<td>Conjecturing</td>
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<td>• Know set notation</td>
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<td>Algebraic proficiency: visualising I</td>
<td>12</td>
<td>• Calculate volumes of spheres, cones and pyramids</td>
<td>• Know the conventions for representing inequalities graphically</td>
</tr>
<tr>
<td>Exploring fractions, decimals and percentages</td>
<td>6</td>
<td>• Understand and use vectors</td>
<td>• Know the formulae for the volume of a sphere, a cone and a pyramid</td>
</tr>
<tr>
<td>Solving equations and inequalities III</td>
<td>8</td>
<td>• Analyse data through measures of central tendency, including quartiles</td>
<td>• Know the formulae for the surface area of a sphere, and the curved surface area of a cone</td>
</tr>
<tr>
<td>Understanding risk</td>
<td>6</td>
<td>• Stage 10 BAM Progress Tracker Sheet</td>
<td>• Know the circle theorems</td>
</tr>
<tr>
<td>Analysing statistics</td>
<td>12</td>
<td></td>
<td>• Know the characteristic shape of the graph of an exponential function</td>
</tr>
<tr>
<td>Algebraic proficiency: visualising II</td>
<td>6</td>
<td></td>
<td>• Know the meaning of roots, intercepts and turning points</td>
</tr>
<tr>
<td>Mathematical movement II</td>
<td>4</td>
<td></td>
<td>• Know the definition of acceleration</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>57</strong></td>
<td></td>
<td><strong>Total</strong></td>
</tr>
</tbody>
</table>
### Key concepts
- Make links to similarity (including trigonometric ratios) and scale factors
- Know the exact values of \( \sin \theta \) and \( \cos \theta \) for \( \theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ \) and \( 90^\circ \); know the exact value of \( \tan \theta \) for \( \theta = 0^\circ, 30^\circ, 45^\circ \) and \( 60^\circ \)
- Know the trigonometric ratios, \( \sin \theta = \text{opposite}/\text{hypotenuse} \), \( \cos \theta = \text{adjacent}/\text{hypotenuse} \), \( \tan \theta = \text{opposite}/\text{adjacent} \)
- Apply it to find angles and lengths in right-angled triangles in two dimensional figures

### Possible learning intentions
- Investigate similar triangles
- Explore trigonometry in right-angled triangles
- Set up and solve trigonometric equations
- Use trigonometry to solve practical problems

### Prerequisites
- Understand and work with similar shapes
- Solve linear equations, including those with the unknown in the denominator of a fraction
- Understand and use Pythagoras’ theorem

### Mathematical language
- Similar
- Opposite
- Adjacent
- Hypotenuse
- Trigonometry
- Function
- Ratio
- Sine
- Cosine
- Tangent
- Angle of elevation, angle of depression

**Notation**
- \( \sin \theta \) stands for the ‘sine of \( \theta \)’
- \( \sin^{-1} \) is the inverse sine function, and not \( 1/\sin \theta \)

### Possible success criteria
- Appreciate that the ratio of corresponding sides in similar triangles is constant
- Label the sides of a right-angled triangle using a given angle
- Choose an appropriate trigonometric ratio that can be used in a given situation
- Understand that sine, cosine and tangent are functions of an angle
- Establish the exact values of \( \sin \theta \) and \( \cos \theta \) for \( \theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ \) and \( 90^\circ \)
- Establish the exact value of \( \tan \theta \) for \( \theta = 0^\circ, 30^\circ, 45^\circ \) and \( 60^\circ \)
- Know how to select the correct mode on a scientific calculator
- Use a calculator to find the sine, cosine and tangent of an angle
- Know the trigonometric ratios, \( \sin \theta = \text{opp}/\text{hyp}, \cos \theta = \text{adj}/\text{hyp}, \tan \theta = \text{opp}/\text{adj} \)
- Set up and solve a trigonometric equation to find a missing side in a right-angled triangle
- Set up and solve a trigonometric equation when the unknown is in the denominator of a fraction
- Set up and solve a trigonometric equation to find a missing angle in a right-angled triangle
- Use trigonometry to solve problems involving bearings
- Use trigonometry to solve problems involving an angle of depression or an angle of elevation

### Possible misconceptions
- Some students may think that sine, cosine and tangent are opposite
- The mnemonic ‘Some Of Harry’s Cats Are Heavier Than Other Animals’ is used to help students remember the trigonometric ratios

### Possible opportunities and probing questions
- Show me an angle and its exact sine (cosine / tangent). And another ...
- Convince me that you have chosen the correct trigonometric function
- (When exploring sets of similar triangles and working out ratios in corresponding cases) why do you think that the results are all similar, but not the same? Could we do anything differently to get results that are closer? How could we make a final conclusion for each ratio?

### Suggested activities
- **KM:** From set squares to trigonometry
- **KM:** Trigonometry flowchart
- **NRICH:** Trigonometric protractor
- **NRICH:** Sine and cosine
- **Learning review**
- **KM:** 10M10 BAM Task

### Common approaches
All students explore sets of similar triangles with angles of (at least) 30°, 45° and 60° as an introduction to the three trigonometric ratios. The mnemonic ‘Some Of Harry’s Cats Are Heavier Than Other Animals’ is used to help students remember the trigonometric ratios.
### Calculating 8 hours

#### Key concepts
- estimate powers and roots of any given positive number
- calculate with roots, and with integer and fractional indices
- calculate exactly with surds
- apply and interpret limits of accuracy, including upper and lower bounds

#### The Big Picture: Calculation progression map

<table>
<thead>
<tr>
<th>Possible learning intentions</th>
<th>Possible success criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate with powers and roots</td>
<td>Estimate squares and cubes of numbers up to 100</td>
</tr>
<tr>
<td>Calculate with powers and roots</td>
<td>Estimate powers of numbers up to 10</td>
</tr>
<tr>
<td>Explore the impact of rounding</td>
<td>Estimate square roots of numbers up to 150</td>
</tr>
<tr>
<td></td>
<td>Estimate cube roots of numbers up to 20</td>
</tr>
<tr>
<td></td>
<td>Know that ( a^0 = 1 )</td>
</tr>
<tr>
<td></td>
<td>Know that ( a^{-n} = \frac{1}{a^n} )</td>
</tr>
<tr>
<td></td>
<td>Know that ( a^{\frac{1}{n}} = \sqrt[n]{a} )</td>
</tr>
<tr>
<td></td>
<td>Calculate with negative powers</td>
</tr>
<tr>
<td></td>
<td>Calculate with fractional powers</td>
</tr>
<tr>
<td></td>
<td>Calculate exactly with surds</td>
</tr>
<tr>
<td></td>
<td>Use the functionality of a scientific calculator when calculating with roots and powers</td>
</tr>
<tr>
<td></td>
<td>Choose the required minimum and maximum values when solving a problem involving upper and lower bounds</td>
</tr>
<tr>
<td></td>
<td>Calculate the upper and lower bounds in a given situation</td>
</tr>
</tbody>
</table>

#### Prerequisites
- Calculate with positive indices using written methods and negative indices in the context of standard form
- Know the multiplication and division laws of indices
- Round to a given number of decimal places or significant figures
- Identify the minimum and maximum values of an amount that has been rounded (to nearest \( x \), \( x \) d.p., \( x \) s.f.)

#### Mathematical language
- Power
- Root
- Index, Indices
- Standard form
- Inequality
- Truncate
- Round
- Minimum Bound
- Maximum Bound
- Interval
- Decimal place
- Significant figure
- Surd
- Limit

#### Notation
- Inequalities: e.g. \( x > 3 \), \( -2 < x \leq 5 \)
- \( \sqrt[3]{a^2} \neq \sqrt[3]{a} + \sqrt[3]{b} \)
- \( \sqrt[3]{a} \pm \sqrt[3]{b} = \frac{a + b}{\sqrt[3]{ab}} \) and \( \sqrt[3]{a} \times b = \sqrt[3]{ab} \times \sqrt[3]{b} \)

#### Pedagogical notes
- Surd is derived from the Latin ‘surdus’ ('deaf' or 'mute'). A surd is therefore a number that cannot be expressed ('spoken') as a rational number.
- Calculating with surds includes establishing the rules:
  - \( a^0 = 1 \)
  - \( a^{-n} = \frac{1}{a^n} \)
  - \( a^{\frac{1}{n}} = \sqrt[n]{a} \)

#### Prerequisites
- Mathematical language
- Pattern sniffing is encouraged to establish the result \( a^0 = 1, a^{-n} = \frac{1}{a^n} \), ie \( 2^0 = 2 \times 2 = 2 \)
- \( 2^2 = 2 \times 2 = 4 \)
- \( 2^1 = 2 \)
- \( 2^{\frac{1}{2}} = \frac{1}{2} \)

#### Possible misconceptions
- Some students may think that negative indices change the sign of a number, for example \( 2^{-1} = -2 \) rather than \( 2^{\frac{1}{2}} = \frac{1}{2} \)
- Some students may think \( \sqrt[3]{a + b} = \sqrt[3]{a} \pm \sqrt[3]{b} \)
- Some students may struggle to understand why the maximum bound of a rounded number is actually a value which would not round to that number; i.e. if given the fact that a number 'x' is rounded to 1 decimal place the result is 2.5, they might write '2.45 < x < 2.55'

#### Possible activities
- KM: Maths to Infinity: Standard form
- KM: Maths to Infinity: Indices
- NRICH: Powers and Roots – Short Problems
- NRICH: Power Countdown
- Powers of 10 (external website)

#### Learning review
- www.diagnosticquestions.com

#### Reasoning opportunities and probing questions
- Show me a surd. And another. And another ...
- When a number 'x' is rounded to 1 decimal place the result is 2.5. Jenny writes '2.45 < x < 2.55'. What is wrong with Jenny’s statement? How would you correct it?
- Always/Sometimes/ Never: \( \sqrt[3]{a + b} = \sqrt[3]{a} \pm \sqrt[3]{b} \)
- Convince me that \( 2^3 = \frac{1}{8} \)

#### Possible misconceptions
- Some students may think that negative indices change the sign of a number, for example \( 2^{-1} = -2 \) rather than \( 2^{\frac{1}{2}} = \frac{1}{2} \)
- Some students may think \( \sqrt[3]{a + b} = \sqrt[3]{a} \pm \sqrt[3]{b} \)
- Some students may struggle to understand why the maximum bound of a rounded number is actually a value which would not round to that number; i.e. if given the fact that a number 'x' is rounded to 1 decimal place the result is 2.5, they might write '2.45 < x < 2.55'
# Solving equations and inequalities I

## Key concepts
- find approximate solutions to equations numerically using iteration
- solve two linear simultaneous equations in two variables algebraically

## The Big Picture
**Algebra progression map**

### Possible learning intentions
- Find approximate solutions to complex equations
- Solve problems involving simultaneous equations
- Find approximate solutions to complex equations numerically using iteration
- Solve two linear simultaneous equations in two variables algebraically

### Possible success criteria
- Understand the concept of decimal search to solve a complex equation
- Use decimal search to solve a complex equation
- Understand the process of interval bisection to locate an approximate solution for a complex equation
- Use interval bisection to locate an approximate solution for a complex equation
- Rearrange an equation to form an iterative formula
- Use an iterative formula to find approximate solutions to equations
- Understand the concept of solving simultaneous equations by substitution
- Decide whether to use elimination or substitution to solve a pair of simultaneous equations
- Solve two linear simultaneous equations in two variables by substitution
- Solve two linear simultaneous equations in two variables by elimination (multiplication of both equations required)
- Derive and solve two simultaneous equations in complex cases
- Interpret the solution to a pair of simultaneous equations

### Prerequisites
- Understand the concept of solving simultaneous equations by elimination
- Solve two linear simultaneous equations in two variables in very simple cases (no multiplication required)
- Solve two linear simultaneous equations in two variables in simple cases (multiplication of one equation only required)

### Mathematical language
- **Unknown**
- **Solve**
- **Solution set**
- **Interval**
- **Decimal search**
- **Iteration**
- **Simultaneous equations**
- **Substitution**
- **Elimination**

#### Notation
- (a, b) for an open interval
- [a, b] for a closed interval

### Pedagogical notes
- Pupils have been introduced to solving simultaneous equations using elimination in simple cases in Stage 9. This includes either no multiplication being required or multiplication of just one equation being required. Solving simultaneous equations using substitution is new to this Stage.
- Iteration is introduced as a process for finding approximate solutions to non-linear equations. GCSE examples can be found here.
- NCETM: Departmental workshops: Simultaneous equations
- NCETM: Glossary

### Common approaches
- Pupils are taught to label the equations (1) and (2), and label the subsequent equations (3), (4), etc.
- Pupils use the 'ANS' key on their calculators when finding an approximate solution using iteration

### Reasoning opportunities and probing questions
- Show me a pair of simultaneous equations with a solution $x = 4$, $y = -2$. And another. And another ...
- Convince me $x + 2y = 11$, $3x + 4y = 18$ can be solved using substitution and using elimination. Which method is best in this case?
- Always/Sometimes/ Never: Solving a pair of simultaneous equations using elimination is more efficient than using substitution

### Suggested activities
- KM: Stick on the Maths: ALG2 Simultaneous linear equations
- KM: Convinced?: ALG2 Simultaneous linear equations
- NRICH: Matchless
- AQA: Bridging Units Resource Pocket 4
- Learning review
  - [www.diagnosticquestions.com](http://www.diagnosticquestions.com)

### Possible misconceptions
- Some pupils may not check the solution to a pair of simultaneous equations satisfy both equations
- Some pupils may not multiply all coefficients, or the constant, when multiplying an equation
- Some pupils may struggle to deal with negative numbers correctly when adding or subtracting the equations
### Mathematical movement I

#### 10 hours

**Key concepts**
- identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement (including fractional scale factors)
- make links between similarity and scale factors
- describe the changes and invariance achieved by combinations of rotations, reflections and translations

**The Big Picture:** Position and direction progression map

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### Possible learning intentions

<table>
<thead>
<tr>
<th>Possible learning intentions</th>
<th>Possible success criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Explore enlargement of 2D shapes</td>
<td>- Use the centre and scale factor to carry out an enlargement of a 2D shape with a fractional scale factor</td>
</tr>
<tr>
<td>- Investigate the transformation of 2D shapes</td>
<td>- Find the scale factor of an enlargement with fractional scale factor</td>
</tr>
<tr>
<td></td>
<td>- Find the centre of an enlargement with fractional scale factor</td>
</tr>
<tr>
<td></td>
<td>- Make links between similarity and scale factors</td>
</tr>
<tr>
<td></td>
<td>- Solve problems involving similarity</td>
</tr>
<tr>
<td></td>
<td>- Perform a sequence of transformations on a 2D shape</td>
</tr>
<tr>
<td></td>
<td>- Find and describe a single transformation given two congruent 2D shapes</td>
</tr>
</tbody>
</table>

**Prerequisites**

- Use the centre and scale factor to carry out an enlargement of a 2D shape with a positive integer scale factor
- Use the concept of scaling in diagrams
- Carry out reflection, rotations and translations of 2D shapes

**Mathematical language**

<table>
<thead>
<tr>
<th>Perpendicular bisector</th>
<th>Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Similar</td>
<td>Congruent</td>
</tr>
<tr>
<td>Invariance</td>
<td>Transformation</td>
</tr>
<tr>
<td>Rotation</td>
<td>Reflection</td>
</tr>
<tr>
<td>Translation</td>
<td>Enlargement</td>
</tr>
</tbody>
</table>

**Pedagogical notes**

- Pupils have identified, described and constructed congruent shapes using rotation, reflection and translation in Stage 7. They have also identified, described and constructed similar shapes using enlargement in Stage 8 and experienced enlarging shapes using positive integer scale factors in Stage 9.

**NCETM: Glossary**

**Common approaches**

*All pupils should experience using dynamic software (e.g. Autograph) to explore enlargements using fractional scale factors*

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### Reasoning opportunities and probing questions

<table>
<thead>
<tr>
<th>Reasoning opportunities and probing questions</th>
<th>Suggested activities</th>
<th>Possible misconceptions</th>
</tr>
</thead>
</table>
| - Show me a pair of similar shapes. And another. And another ... | KM: [Stick on the Maths SSM3: Enlargement (fractional scale factor)](https://www.stickonthe.maths.org.uk/3-enlargements/fractional-scale-factor)  
KM: [Stick on the Maths SSM1: Congruence and similarity](https://www.stickonthe.maths.org.uk/1-congruence-similarity)  
NRICH: [Growing Rectangles](https://nrich.maths.org/1196)  
Learning review [www.diagnosticquestions.com](https://www.diagnosticquestions.com) | - Some pupils may think that the resulting image of an enlargement has to be larger than the original object.  
- Some pupils may think that the order of transforming an object does not have an effect on the size and position of the final image.  
- Some pupils may link scale factors and similarity using an additive, rather than multiplicative, relationship. |
### Algebraic proficiency: tinkering

**Key concepts**
- simplify and manipulate algebraic expressions involving algebraic fractions
- manipulate algebraic expressions by expanding products of more than two binomials
- simplify and manipulate algebraic expressions (including those involving surds) by expanding products of two binomials and factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares
- manipulate algebraic expressions by factorising quadratic expressions of the form $ax^2 + bx + c$

**Possible learning intentions**
- Manipulate algebraic expressions
- Investigate inverse functions

**Possible success criteria**
- Add (subtract, multiply, divide) algebraic fractions
- Simplify an algebraic fraction
- Identify when it is necessary to find two linear expressions to factorise a quadratic expression
- Expand the product of two binomials involving surds
- Factorise an expression involving the difference of two squares
- Factorise a quadratic expression of the form $ax^2 + bx + c$
- Identify when it is necessary to factorise the numerator and/or denominator in order to simplify an algebraic fraction
- Simplify an algebraic fraction that involves factorisation

**Possible misconceptions**
- The difference of two squares is explained using visual representations. Pupils manipulate a Pictorial representation of the difference of two squares (monic method) to help them understand the concept. This method uses algebra tiles to explore factoring quadratics. Teachers also need to help pupils 'see' the difference of two squares by using pictorial representations.

**Prerequisites**
- Calculate with negative numbers
- Multiply two linear expressions of the form $(x \pm a)(x \pm b)$
- Factorise a quadratic expression of the form $x^2 + bx + c$
- Add, subtract, multiply and divide proper fractions

**Mathematical language**
- Equivalent
- Equation
- Expression
- Expand
- Linear
- Quadratic
- Algebraic Fraction
- Difference of two squares
- Binomial
- Factorise
- Notation

**Pedagogical notes**
- Pupils have applied the four operations to proper, and improper, fractions in Stage 7 and factorised quadratics of the form $x^2 + bx + c$ in Stage 9. Pupils should build on the experiences of using the grid method in Stage 9 to expand products of more than two binomials. Eg $(x + 2)(x + 3)(x + 4) = (x^2 + 5x + 6)(x - 4) = x^3 + x^2 - 14x - 24$

<table>
<thead>
<tr>
<th>Expression</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^3$</td>
<td>$+5x$</td>
</tr>
<tr>
<td>$x^2$</td>
<td>$+6$</td>
</tr>
<tr>
<td>$-4x^2$</td>
<td>$-20x$</td>
</tr>
</tbody>
</table>

- Teachers also need to help pupils 'see' the difference of two squares by using pictorial representations.

**Common approaches**
- Use the 'monic method' for factorising quadratics of the form $ax^2 + bx + c$
- Pupils manipulate algebra tiles to explore factoring quadratics.
- The difference of two squares is explained using visual representation.

**Reasoning opportunities and probing questions**
- The answer is $2x^2 + 10x + c$. Show me a possible question. And another.
- Kenny simplifies $\frac{2x^2 + 5x + 2}{2x + 1}$ as $x + 2$. Do you agree with Kenny? Explain your answer.
- Convince me that $103^2 - 97^2 = 1200$ without a calculator.
- Convince me that $4x^2 - 9 = (3x - 2)(3x + 2)$.
- Jenny thinks that $(3x - 2)^2 = 3x^2 + 12x + 4$. Do you agree with Jenny? Explain your answer.
- Convince me that $\frac{2x^4 + 5x^2}{2x + 1} = x + 2$

**Suggested activities**
- KM: Maths to Infinity: Brackets
- KM: Maths to Infinity: Quadratics
- KM: Stick on the Maths: Quadratic sequences
- NRICH: What’s possible?
- NRICH: Finding Factors
- Algebra Tiles (external site)
- Learning review
  - www.diagnosticquestions.com

**Possible misconceptions**
- Once pupils know how to factorise a quadratic expression of the form $x^2 + bx + c$ they might overcomplicate the simpler case of factorising an expression such as $3x^2 + 6x = (3x + 2)(x + 2)$.
- Some pupils may think that $(x + a)^2 = x^2 + a^2$.
- Some pupils may apply the ‘rules of factorising’ quadratics of the form $x^2 + bx + c$ to quadratics of the form $ax^2 + bx + c$ e.g. $2x^2 + 7x + 10 \equiv (2x + 5)(x + 2)$ because $2 \times 5 = 10$ and $2 + 5 = 7$. 

www.kangaroomaths.com
### Key concepts
- Interpret equations that describe direct and inverse proportion
- Recognise and interpret graphs that illustrate direct and inverse proportion
- Understand that X is inversely proportional to Y is equivalent to X is proportional to 1/Y

### Prerequisites
- Know the difference between direct and inverse proportion
- Recognise direct or inverse proportion in a situation
- Know the features of a graph that represents a direct or inverse proportion situation
- Know the features of an expression (or formula) that represents a direct or inverse proportion situation
- Understand the connection between the multiplier, the expression and the graph

### Possible learning intentions
- Explore differences between direct and inverse proportion
- Investigate ways of representing proportion in situation
- Solve problems involving proportion

### Possible success criteria
- Recognise a graph that illustrates direct proportion
- Recognise a graph that illustrates inverse proportion
- Interpret a graph that illustrates direct proportion
- Interpret a graph that illustrates inverse proportion
- Understand that X is inversely proportional to Y is equivalent to X is proportional to 1/Y
- Interpret equations that describe direct proportion
- Interpret equations that describe inverse proportion
- Solve problems which include finding the multiplier in a situation involving direct proportion
- Solve problems which include finding the multiplier in a situation involving inverse proportion

### Prerequisites
- Mathematical language
- Pedagogical notes

<table>
<thead>
<tr>
<th>Prerequisites</th>
<th>Mathematical language</th>
<th>Pedagogical notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Know the difference between direct and inverse proportion</td>
<td>Direct proportion</td>
<td>Pupils have solved simple problems involving direct and inverse proportion in Stage 9. This unit focuses on developing a formal algebraic approach, including the use of proportionality constants, to solve direct and inverse proportion problems.</td>
</tr>
<tr>
<td>Recognise direct or inverse proportion in a situation</td>
<td>Inverse proportion</td>
<td>NCETM: Departmental workshops: Proportional Reasoning</td>
</tr>
<tr>
<td>Know the features of a graph that represents a direct or inverse proportion situation</td>
<td>Multiplier</td>
<td>NCETM: Glossary</td>
</tr>
<tr>
<td>Know the features of an expression (or formula) that represents a direct or inverse proportion situation</td>
<td>Notation</td>
<td></td>
</tr>
<tr>
<td>Understand the connection between the multiplier, the expression and the graph</td>
<td>( \propto ) - ‘proportional to’</td>
<td>Common approaches</td>
</tr>
</tbody>
</table>

### Reasoning opportunities and probing questions

- **Show me an example of two quantities that will be in direct proportion. And another. And another ...**
- **Convince me that this information shows a proportional relationship. What type of proportion is it?**
  - 40 50
  - 60 75
  - 80 100
- **Always/Sometimes/Never: X is inversely proportional to Y is equivalent to X is proportional to 1/Y**

### Suggested activities

- KM: Graphing proportion
- KM: Stick on the Maths NNS1: Understanding Proportionality
- KM: Stick on the Maths CALC1: Proportional Change and multiplicative methods
- KM: Convinced: NNS1: Understanding Proportionality
- KM: Convinced: CALC1: Proportional Change and multiplicative methods
- NRICH: In Proportion
- Learning review
  - www.diagnosticquestions.com

### Possible misconceptions

- Some pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to solve problems
- Some pupils may interpret ‘x is inversely proportional to y’ as y = x/k rather than y = k/x
- Some pupils may think that the proportionality constant always has to be greater than 1
### Key concepts
- deduce expressions to calculate the nth term of quadratic sequences
- recognise and use simple geometric progressions (r^n where n is an integer, and r is a rational number > 0)

### Possible learning intentions
- Explore quadratic sequences
- Investigate geometric progressions

### Possible success criteria
- Understand the meaning of a quadratic sequence
- Find the term in \(x^2\) for a quadratic sequence
- Find the nth term of a sequence of the form \(ax^2\)
- Find the nth term of a sequence of the form \(ax^2 + b\)
- Find the nth term of a sequence of the form \(ax^2 + bx + c\)
- Understand the difference between an arithmetic progression, a quadratic sequence and a geometric progression
- Recognise a simple geometric progression
- Find the next three terms in a geometric progression
- Find a given term in a simple geometric progression
- Describe a geometric progression

### Possible misconceptions
- Some students may think that it is possible to find an nth term for any sequence.
- Some students may think that the second difference (of a quadratic sequence) is equivalent to the coefficient of \(x^2\).
- Some students may substitute into \(ax^2\) incorrectly, working out \((ax)^2\) instead.

### Prerequisites
- Find the nth term for an increasing linear sequence
- Find the nth term for a decreasing linear sequence
- Identify quadratic sequences
- Establish the first and second differences of a quadratic sequence
- Find the next three terms in a quadratic sequence

### Mathematical language
<table>
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<tr>
<th>Term</th>
<th>Notation</th>
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<tbody>
<tr>
<td>nth term</td>
<td>(T(n)) is often used to indicate the ‘nth term’</td>
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<tr>
<td>Generate</td>
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### Reasoning opportunities and probing questions
- Show me a geometric progression. And another. And another....
- Show me a quadratic sequence with nth term \(3x^2 + bx + c\). And another. And another....
- Convince me the nth term of 19, 16, 14, ... is \(20 - x^2\).
- Kenny thinks 1, 1, 1, 1, 1, ... is an arithmetic sequence. Jenny thinks 1, 1, 1, 1, 1, ... is a geometric sequence. Who is correct? Explain your answer.

### Suggested activities
- KM: Maths to Infinity: Sequences
- KM: Stick on the Maths: Quadratic sequences
- NRICH: Growing Surprises
- Learning review: [www.diagnosticquestions.com](http://www.diagnosticquestions.com)
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## Calculating space

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### Prerequisites
- Mathematical language

### Reasoning opportunities and probing questions
- Suggested activities

### The Big Picture: Measurement and mensuration progression map
- calculate surface area and volume of spheres, pyramids, cones and composite solids
- apply the concepts of congruence and similarity, including the relationships between length, areas and volumes in similar figures

### Possible misconceptions
- Suggested activities

www.kangaroomaths.com
## Conjecturing

### Key concepts
- apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results

### The Big Picture: Properties of Shape progression map

### Possible learning intentions
- Investigate geometric patterns using circles
- Explore circle theorems
- Make and prove conjectures

### Possible success criteria
- Know the conditions for creating a right angle with three points on the circumference of a circle
- Know that ‘the angle in a semicircle is a right angle’ (and others – see pedagogical notes)
- Form a conjecture from a geometrical situation
- Set up a proof
- Create a chain of logical steps to create a proof in a geometrical situation
- Identify when a circle theorem can be used to help solve a geometrical problem
- Use a combination of known and derived facts to solve a geometrical problem

### Return to overview

### Bring on the Maths: GCSE Higher Shape

#### Investigating angles in circles: #1, #2, #3, #4

### Prerequisites
- Know the vocabulary of circles
- Know angle facts including angles at a point, on a line and in a triangle
- Know angle facts involving parallel lines and vertically opposite angles
- Know the properties of special quadrilaterals

### Mathematical language
- Radius, radii
- Tangent
- Chord
- Theorem
- Conjecture
- Derive
- Prove, proof
- Counterexample

### Notation
- Notation for equal lengths and parallel lines
- The ‘implies that’ symbol (⇒)

### Pedagogical notes
- Students should also explore the following (paraphrased) circle theorems:
  - Cyclic Quadrilateral: GSP, Word
  - Radius and Tangent: GSP, Word
  - Radius and chord:
  - Angles in the Same Segment: GSP, Word
  - The Angle in the Centre: GSP, Word
  - Two Tangents: GSP, Word
  - Alternate Segment Theorem: GSP, Word

### Common approaches

All students are first introduced to the idea of circle theorems by investigating Thales Theorem. This is then extended to demonstrate that ‘the angle at the centre is twice the angle at the circumference’

All students are given the opportunity to create and explore dynamic diagrams of different circle theorems.

### Reasoning opportunities and probing questions
- How can you use a set square to find the centre of a circle?
- Show me a radius of this circle. And another, and another … (What does this tell you about the lengths? About the triangle?)
- Provide the steps for a geometrical proof of a circle theorem and ask students to ‘unjumble’ them and create the proof, explaining their thinking at each step
- Use the ‘Always / Sometimes / Never’ approach to introduce a circle theorem

### Suggested activities
- KM: Right angle challenge
- KM: Thales Theorem
- KM: 6 point circles, 8 point circles, 12 point circles
- KM: Dynamic diagrams
- NRICH: Circle theorems
- Hwb: Cadair Idris
- Hwb: Cyclic quadrilaterals

### Learning review
- www.diagnosticquestions.com

### Possible misconceptions
- Some students may think that a cyclic quadrilateral is formed using three points on the circumference along with the centre of the circle
- Some students may not appreciate the significance of standard geometrical notation for equal lengths and angles, and think that lengths / angles are equal ‘because they look equal’
- Some students may not realise that they can extend the lines on diagrams to help establish necessary facts
### Key concepts

- plot and interpret graphs (including exponential graphs) and graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
- calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs), and interpret results in cases such as distance-time graphs, velocity-time graphs and graphs in financial contexts
- interpret the gradient at a point on a curve as the instantaneous rate of change
- identify and interpret roots, intercepts, turning points of quadratic functions graphically

### Possible learning intentions

### Possible success criteria

### Prerequisites

### Mathematical language

### Pedagogical notes

### Reasoning opportunities and probing questions

### Suggested activities

### Possible misconceptions
### Exploring fractions, decimals and percentages

- **3 hours**

#### Key concepts
- change recurring decimals into their corresponding fractions and vice versa
- set up, solve and interpret the answers in growth and decay problems, including compound interest

#### The Big Picture:
- Fractions, decimals and percentages progression map

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[Return to overview](#)
### Solving equations and inequalities III

**8 hours**

**Key concepts**
- solve quadratic equations algebraically by factorising
- solve quadratic equations (including those that require rearrangement) algebraically by factorising
- find approximate solutions to quadratic equations using a graph
- deduce roots of quadratic functions algebraically

**The Big Picture:** [Algebra progression map](#)

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Key concepts
- apply systematic listing strategies including use of the product rule for counting
- calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams.

The Big Picture: Probability progression map
## Analysing statistics

### Key concepts
- Infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling
- Construct and interpret diagrams for grouped discrete data and continuous data, i.e. cumulative frequency graphs, and know their appropriate use
- Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data, including box plots
- Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency including quartiles and inter-quartile range

### Possible learning intentions

### Possible success criteria

### Prerequisites

### Mathematical language

### Pedagogical notes

### Reasoning opportunities and probing questions

### Suggested activities

### Possible misconceptions
### Algebraic proficiency: visualising II

#### Key concepts
- use the form $y = mx + c$ to identify perpendicular lines
- recognise and use the equation of a circle with centre at the origin
- find the equation of a tangent to a circle at a given point

#### The Big Picture: Algebra progression map

#### Possible learning intentions

#### Possible success criteria

#### Prerequisites

#### Mathematical language

#### Pedagogical notes

#### Reasoning opportunities and probing questions

#### Suggested activities

#### Possible misconceptions
### Mathematical movement II

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**5 hours**

**Key concepts**
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors

| The Big Picture: Position and direction progression map |

**Possible learning intentions**

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