### Mathematics overview: Stage 8

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<td>• Apply the four operations with negative numbers</td>
<td>• Know how to write a number as a product of its prime factors</td>
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<tr>
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<td>• Convert numbers into standard form and vice versa</td>
<td>• Know how to round to significant figures</td>
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<tr>
<td>Visualising and constructing</td>
<td>8</td>
<td>• Apply the multiplication, division and power laws of indices</td>
<td>• Know the order of operations including powers</td>
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<tr>
<td>Understanding risk I</td>
<td>6</td>
<td>• Convert between terminating decimals and fractions</td>
<td>• Know how to enter negative numbers into a calculator</td>
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<tr>
<td>Algebraic proficiency: tinkering</td>
<td>10</td>
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<td>• Know that ( a^0 = 1 )</td>
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<tr>
<td>Exploring fractions, decimals and percentages</td>
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<td>Proportional reasoning</td>
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<tr>
<td>Calculating fractions, decimals and percentages</td>
<td>6</td>
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<td>• Know how to identify corresponding angles</td>
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<tr>
<td>Solving equations and inequalities</td>
<td>4</td>
<td>• Plot and interpret graphs of linear functions</td>
<td>• Know how to find the angle sum of any polygon</td>
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<tr>
<td>Calculating space</td>
<td>9</td>
<td>• Apply the formulae for circumference and area of a circle</td>
<td>• Know that circumference = ( 2\pi r = \pi d )</td>
</tr>
<tr>
<td>Algebraic proficiency: visualising</td>
<td>9</td>
<td>• Calculate theoretical probabilities for single events</td>
<td>• Know that area of a circle = ( \pi r^2 )</td>
</tr>
<tr>
<td>Understanding risk II</td>
<td>5</td>
<td>• Stage 8 BAM Progress Tracker Sheet</td>
<td>• Know that volume of prism = area of cross-section × length</td>
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<tr>
<td>Presentation of data</td>
<td>4</td>
<td></td>
<td>• Know to use the midpoints of groups to estimate the mean of a set of grouped data</td>
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<tr>
<td>Measuring data</td>
<td>6</td>
<td></td>
<td>• Know that probability is measured on a 0-1 scale</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>• Know that the sum of all probabilities for a single event is 1</td>
</tr>
</tbody>
</table>
### Key concepts
- use the concepts and vocabulary of prime numbers, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem
- round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures)
- interpret standard form \( A \times 10^n \), where \( 1 \leq A < 10 \) and \( n \) is an integer

### Possible learning intentions
- Identify and use the prime factorisation of a number
- Understand and use standard form

### Prerequisites
- Know the meaning of a prime number
- Recall prime numbers up to 50
- Understand the use of notation for powers
- Know how to round to the nearest whole number, 10, 100, 1000 and to decimal places
- Multiply and divide numbers by powers of 10
- Know how to identify the first significant figure in any number
- Approximate by rounding to the first significant figure in any number

### Mathematical language
- Prime
- Prime factor
- Prime factorisation
- Product
- Venn diagram
- Highest common factor
- Lowest common multiple
- Standard form
- Significant figure

**Notation**
Index notation: e.g. \( 5^3 \) is read as ‘5 to the power of 3’ and means ‘3 lots of 5 multiplied together’

Standard form (see key concepts) is sometimes called ‘standard index form’, or more properly, ‘scientific notation’

### Pedagogical notes
- Pupils should explore the ways to enter and interpret numbers in standard form on a scientific calculator. Different calculators may vary well have different displays, notations and methods.
- Liaise with the science department to establish when students first meet the use of standard form, and in what contexts they will be expected to interpret it.
- NRICH: [Divisibility testing](https://nrich.maths.org/5483)
- NCETM: [Glossary](https://www.ncetm.org.uk/)

### Common approaches
*The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.*

### Possible misconceptions
- Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as ‘a number which is divisible by itself and 1’
- Some pupils may think 35 934 = 36 to two significant figures
- When converting between ordinary and standard form some pupils may incorrectly connect the power to the number of zeros; e.g. \( 4 \times 10^5 = 400 000 \) so \( 4.2 \times 10^5 = 4 200 000 \)
- Similarly, when working with small numbers (negative powers of 10) some pupils may think that the power indicates how many zeros should be placed between the decimal point and the first non-zero digit.

### Reasoning opportunities and probing questions
- Show me two (three-digit) numbers with a highest common factor of 18. And another. And another...
- Show me two numbers with a lowest common multiple of 240. And another. And another...
- Jenny writes \( 7.1 \times 10^{-5} = 0.0000071 \). Kenny writes \( 7.1 \times 10^{-5} = 0.000071 \). Who do you agree with? Give reasons for your answer.

### Suggested activities
- KM: [Astronomical numbers](https://www.kangaroornaths.com/astronomical-numbers)
- KM: [Interesting standard form](https://www.kangaroornaths.com/interesting-standard-form)
- KM: [Powers of ten](https://www.kangaroornaths.com/powers-of-ten)
- KM: [Powers of ten](https://www.kangaroornaths.com/powers-of-ten) film (external site)
- KM: [The scale of the universe](https://www.kangaroornaths.com/the-scale-of-the-universe) animation (external site)

### Learning review
- KM: [8M2 BAM Task](https://www.kangaroornaths.com/8m2-bam-task)
### Calculating

#### Key concepts
- apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative
- use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

#### 9 hours

| The Big Picture: Calculation progression map
|

#### Possible learning intentions
- Calculate with negative numbers
- Apply the correct order of operations

#### Possible success criteria
- Add or subtract from a negative number
- Add (or subtract) a negative number to (from) a positive number
- Add (or subtract) a negative number to (from) a negative number
- Multiply with negative numbers
- Divide with negative numbers
- Know how to square (or cube) a negative number
- Substitute negative numbers into expressions
- Enter negative numbers into a calculator
- Interpret a calculator display when working with negative numbers
- Understand how to use the order of operations including powers
- Understand how to use the order of operations including roots

#### Prerequisites
- Fluently recall and apply multiplication facts up to 12 × 12
- Know and use column addition and subtraction
- Know the formal written method of long multiplication
- Know the formal written method of short division
- Apply the four operations with fractions and mixed numbers
- Convert between an improper fraction and a mixed number
- Know the order of operations for the four operations and brackets

#### Mathematical language
- **Negative number**
- **Directed number**
- **Improper fraction**
- **Top-heavy fraction**
- **Mixed number**
- **Operation**
- **Inverse**
- **Long multiplication**
- **Short division**
- **Power**
- **Indices**
- **Roots**

#### Pedagogical notes
- Pupils need to know how to enter negative numbers into their calculator and how to interpret the display.
- The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements.
- NRICH: Adding and subtracting positive and negative numbers
- NRICH: History of negative numbers
- NCETM: Departmental workshop: Operations with Directed Numbers
- NCETM: Glossary

#### Common approaches
- Teachers use the language ‘negative number’, and not ‘minus number’, to avoid confusion with calculations.
- Every classroom has a negative number washing line on the wall.
- Long multiplication and short division are to be promoted as the ‘most efficient methods’.
- If any acronym is promoted to help remember the order of operations, then BIDMAS is used as the I stands for indices.

#### Reasoning opportunities and probing questions
- Convince me that \(-3 + (-7) = 4\)
- Show me an example of a calculation involving addition of two negative numbers and the solution -10. And another. And another ...
- Create a Carroll diagram with ‘addition’, ‘subtraction’ as the column headings and ‘one negative number’, ‘two negative numbers’ as the row headings. Ask pupils to create (if possible) a calculation that can be placed in each of the four positions. If they think it is not possible, explain why. Repeat for multiplication and division.

#### Suggested activities
- KM: Summing up
- KM: Developing negatives
- KM: Sorting calculations
- KM: Maths to Infinity: Directed numbers
- Standards Unit: N9 Evaluating directed number statements
- NRICH: Working with directed numbers
- Learning review
- KM: BM1 BAM Task

#### Possible misconceptions
- Some pupils may use a rule stated as ‘two minuses make a plus’ and make many mistakes as a result; e.g. \(-4 + (-6) = 10\)
- Some pupils may incorrectly apply the principle of commutativity to subtraction; e.g. \(4 - (-7) = 3\)
- The order of operations is often not applied correctly when squaring negative numbers. As a result pupils may think that \(x^2 = -9\) when \(x = -3\). The fact that a calculator applies the correct order means that \((-3)^2 = 9\) and this can actually reinforce the misconception. In this situation brackets should be used as follows: \((-3)^2 = 9\).
**Key concepts**
- measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
- identify, describe and construct similar shapes, including on coordinate axes, by considering enlargement
- interpret plans and elevations of 3D shapes
- use scale factors, scale diagrams and maps

**The Big Picture:** Properties of Shape progression map

### Possible learning intentions
- Explore enlargement of 2D shapes
- Use and interpret scale drawings
- Use and interpret bearings
- Explore ways of representing 3D shapes

### Possible success criteria
- Know the vocabulary of enlargement
- Find the centre of enlargement
- Find the scale factor of an enlargement
- Use the centre and scale factor to carry out an enlargement with positive integer (fractional) scale factor
- Know and understand the vocabulary of plans and elevations
- Interpret plans and elevations
- Use the concept of scaling in diagrams
- Measure and state a specified bearing
- Construct a scale diagram involving bearings
- Use bearings to solve geometrical problems

### Prerequisites
- Use a protractor to measure angles to the nearest degree
- Use a ruler to measure lengths to the nearest millimetre
- Understand coordinates in all four quadrants
- Work out a multiplier given two numbers
- Understand the concept of an enlargement (no scale factor)

### Mathematical language
- Similar, Similarity
- Enlarge, enlargement
- Scaling
- Scale factor
- Centre of enlargement
- Object
- Image
- Scale drawing
- Bearing
- Plan, Elevation

**Notation**
Bearings are always given as three figures; e.g. 025°.
Cartesian coordinates: separated by a comma and enclosed by brackets

### Pedagogical notes
- Describing enlargement as a ‘scaling’ will help prevent confusion when dealing with fractional scale factors
- NCETM: Departmental workshops: Enlargement
- NCETM: Glossary
- Common approaches
  - All pupils should experience using dynamic software (e.g. Autograph) to visualise the effect of moving the centre of enlargement, and the effect of varying the scale factor.

### Reasoning opportunities and probing questions
- Give an example of a shape and its enlargement (e.g. scale factor 2) with the guidelines drawn on. How many different ways can the scale factor be derived?
- Show me an example of a sketch where the bearing of A from B is between 90° and 180°. And another. And another ...
- The bearing of A from B is ‘x’. Find the bearing of B from A in terms of ‘x’. Explain why this works.
- Provide the plan and elevations of shapes made from some cubes. Challenge pupils to build the shape and place it in the correct orientation.

### Suggested activities
- KM: [Outdoor Leisure 13](#)
- KM: [Airports and hilltops](#)
- KM: [Plans and elevations](#)
- KM: [Transformation template](#)
- KM: [Enlargement I](#)
- KM: [Enlargement II](#)
- KM: [Investigating transformations](#) with Autograph (enlargement and Main Event II). [Dynamic example](#).
- WisWeb applet: [Building houses](#)
- NRICH: [Who’s the fairest of them all?](#)
- Learning review [www.diagnosticquestions.com](#)

### Possible misconceptions
- Some pupils may think that the centre of enlargement always has to be (0,0), or that the centre of enlargement will be in the centre of the object shape.
- If the bearing of A from B is ‘x’, then some pupils may think that the bearing of B from A is ‘180 – x’.
- The north elevation is the view of a shape from the north (the north face of the shape), not the view of the shape while facing north.
# Understanding risk I

## Key concepts
- relate relative expected frequencies to theoretical probability, using appropriate language and the 0 - 1 probability scale
- record and describe the frequency of outcomes of probability experiments using tables
- construct theoretical possibility spaces for single experiments with equally likely outcomes and use these to calculate theoretical probabilities
- apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one

## Possible learning intentions
- Understand the meaning of probability
- Explore experiments and outcomes
- Develop understanding of probability

## Possible success criteria
- Know that probability is a way of measuring likeliness
- Know and use the vocabulary of probability
- Understand the use of the 0-1 scale to measure probability
- Assess likeliness and place events on a probability scale
- List all the outcomes for an experiment
- Identify equally likely outcomes
- Work out theoretical probabilities for events with equally likely outcomes
- Know how to represent a probability
- Recognise when it is not possible to work out a theoretical probability for an event
- Know that the sum of probabilities for all outcomes is 1
- Apply the fact that the sum of probabilities for all outcomes is 1

## Prerequisites
- Understand the equivalence between fractions, decimals and percentages
- Compare fractions, decimals or percentages
- Simplify a fraction by cancelling common factors

## Mathematical language
- Probability, Theoretical probability
- Event
- Outcome
- Impossible, Unlikely, Evens chance, Likely, Certain
- Equally likely
- Mutually exclusive
- Exhaustive
- Possibility space
- Experiment

### Notation
- Probabilities are expressed as fractions, decimals or percentage. They should not be expressed as ratios (which represent odds) or as words

## Pedagogical notes
- This is the first time students will meet probability.
- It is not immediately apparent how to use words to label the middle of the probability scale. ‘Evens chance’ is a common way to do so, although this can be misleading as it could be argued that there is an even chance of obtaining any number when rolling a fair die.
- NRICH: Introducing probability
- NRICH: Why Do People Find Probability Unintuitive and Difficult?
- NCETM: Glossary

### Common approaches
Every classroom has a display of a probability scale labeled with words and numbers. Pupils create events and outcomes that are placed on this scale.

## Reasoning opportunities and probing questions
- Show me an example of an event and outcome with a probability of 0. And another. And another...
- Always / Sometimes / Never: If I pick a card from a pack of playing cards then the probability of picking a club is ¼
- Label this (eight-sided) spinner so that the probability of scoring a 2 is ¼. How many different ways can you label it?

## Suggested activities
- KM: Probability scale and slideshow version
- KM: Probability loop cards
- NRICH: Dice and spinners interactive
- Learning review
- KM: 8M13 BAM Task

## Possible misconceptions
- Some pupils will initially think that, for example, the probability of it raining tomorrow is ½ as it either will or it won’t.
- Some students may write a probability as odds (e.g. 1:6 or ‘1 to 6’). There is a difference between probability and odds, and therefore probabilities must only be written as fractions, decimals or percentages.
- Some pupils may think that, for example, if they flip a fair coin three times and obtain three heads, then it must be more than likely they will obtain a head next.
## Algebraic proficiency: tinkering

### Key concepts
- use and interpret algebraic notation, including: $a^2 b$ in place of $a \times a \times b$, coefficients written as fractions rather than as decimals
- understand and use the concepts and vocabulary of factors
- simplify and manipulate algebraic expressions by taking out common factors and simplifying expressions involving sums, products and powers, including the laws of indices
- substitute numerical values into scientific formulae
- rearrange formulae to change the subject

### Possible learning intentions
- Understand the notation of algebra
- Manipulate algebraic expressions
- Evaluate algebraic statements

### Possible success criteria
- Know how to write products algebraically
- Use fractions when working in algebraic situations
- Identify common factors (numerical and algebraic) of terms in an expression
- Factorise an expression by taking out common factors
- Simplify an expression involving terms with combinations of variables (e.g. $3a^2b + 4ab + 2a^2 - a^2b$)
- Know the multiplication (division, power, zero) law of indices
- Understand that negative powers can arise
- Substitute positive and negative numbers into formulae
- Be aware of common scientific formulae
- Know the meaning of the ‘subject’ of a formula
- Change the subject of a formula when one step is required
- Change the subject of a formula when two steps are required

### Prerequisites
- Know basic algebraic notation (the rules of algebra)
- Simplify an expression by collecting like terms
- Know how to multiply a single term over a bracket
- Substitute positive numbers into expressions and formulae
- Calculate with negative numbers

### Mathematical language
- **Product**
- **Variable**
- **Term**
- **Coefficient**
- **Common factor**
- **Factorise**
- **Power**
- **Indices**
- **Formula, Formulae**
- **Subject**
- **Change the subject**

### Notation
- See key concepts above

### Reasoning opportunities and probing questions
- Convince me $a^0 = 1$.
- What is wrong with this statement and how can it be corrected: $5^2 \times 5^4 = 5^8$ ?
- Jenny thinks that if $y = 2x + 1$ then $x = (y - 1)/2$. Kenny thinks that if $y = 2x + 1$ then $x = y/2 - 1$. Who do you agree with? Explain your thinking.

### Suggested activities
- KM: Missing powers
- KM: Laws of indices - Some useful questions.
- KM: Maths to Infinity: Indices
- KM: Scientific substitution (Note that page 2 is hard)
- NRICH: Temperature

### Possible misconceptions
- Some pupils may misapply the order of operation when changing the subject of a formula
- Many pupils may think that $a^0 = 0$
- Some pupils may not consider $4ab$ and $3ba$ as ‘like terms’ and therefore will not ‘collect’ them when simplifying expressions

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### Pedagogical notes

During this unit pupils should experience factorising a quadratic expression such as $6x^2 + 2x$. Collaborate with the science department to establish a list of formulae that will be used, and ensure consistency of approach and experience.

NCETM: Algebra
NCETM: Departmental workshop: Index Numbers
NCETM: Departmental workshops: Deriving and Rearranging Formulae
NCETM: Glossary

### Common approaches

*Once the laws of indices have been established, all teachers refer to ‘like numbers multiplied, add the indices’ and ‘like numbers divided, subtract the indices. They also generalise to $a^m \times a^n = a^{m+n}$, etc.*

*When changing the subject of a formula the principle of balancing (doing the same to both sides) must be used rather than a ‘change side, change sign’ approach.*

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Visit www.kangaroomaths.com for more resources.
### Exploring fractions, decimals and percentages

**3 hours**

**Key concepts**
- work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and 7/2 or 0.375 or 3/8)

**The Big Picture:** [Fractions, decimals and percentages progression map](#)

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<table>
<thead>
<tr>
<th>Possible learning intentions</th>
<th>Possible success criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Explore links between fractions, decimals and percentages</td>
<td>- Identify if a fraction is terminating or recurring</td>
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<tr>
<td></td>
<td>- Recall some decimal and fraction equivalents (e.g. tenths, fifths, eighths)</td>
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<tr>
<td></td>
<td>- Write a decimal as a fraction</td>
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<td></td>
<td>- Write a fraction in its lowest terms by cancelling common factors</td>
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<tr>
<td></td>
<td>- Identify when a fraction can be scaled to tenths or hundredths</td>
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<tr>
<td></td>
<td>- Convert a fraction to a decimal by scaling (when possible)</td>
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<tr>
<td></td>
<td>- Use a calculator to change any fraction to a decimal</td>
</tr>
<tr>
<td></td>
<td>- Write a decimal as a percentage</td>
</tr>
<tr>
<td></td>
<td>- Write a fraction as a percentage</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Prerequisites</th>
<th>Mathematical language</th>
<th>Pedagogical notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Understand that fractions, decimals and percentages are different ways of representing the same proportion</td>
<td>Fraction, Mixed number, Top-heavy fraction, Percentage, Decimal, Proportion, Terminating, Recurring, Simplify, Cancel</td>
<td>The diagonal fraction bar (solidus) was first used by Thomas Twining (1718) when recorded quantities of tea. The division symbol (÷) is called an obelus, but there is no name for a horizontal fraction bar.</td>
</tr>
<tr>
<td>- Convert between mixed numbers and top-heavy fractions</td>
<td>Notation: Diagonal and horizontal fraction bar</td>
<td>NRICH: <a href="#">History of fractions</a></td>
</tr>
<tr>
<td>- Write one quantity as a fraction of another</td>
<td></td>
<td>NRICH: <a href="#">Teaching fractions with understanding</a></td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Reasoning opportunities and probing questions</th>
<th>Suggested activities</th>
<th>Possible misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Without using a calculator, convince me that 3/8 = 0.375</td>
<td>KM: <a href="#">FDP conversion</a>, Templates for taking notes. KM: <a href="#">Fraction sort</a>, Tasks one and two only. KM: <a href="#">Maths to Infinity: Fractions, decimals, percentages, ratio, proportion</a></td>
<td>- Some pupils may make incorrect links between fractions and decimals such as thinking that 1/5 = 0.15</td>
</tr>
<tr>
<td>- Show me a fraction / decimal / percentage equivalent. And another. And another ...</td>
<td>KM: <a href="#">Matching fractions, decimals and percentages</a></td>
<td>- Some pupils may think that 5% = 0.5, 4% = 0.4, etc.</td>
</tr>
<tr>
<td>- What is the same and what is different: 2.5, 25%, 0.025, %?</td>
<td>KM: <a href="#">BAM BAM Task</a></td>
<td>- Some pupils may think it is not possible to have a percentage greater than 100%.</td>
</tr>
</tbody>
</table>

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NRICH: [Matching fractions, decimals and percentages](#)

NCETM: [Glossary](#)

KM: [FDP conversion](#), Templates for taking notes.

KM: [Fraction sort](#), Tasks one and two only.

KM: [Maths to Infinity: Fractions, decimals, percentages, ratio, proportion](#)

KM: [Matching fractions, decimals and percentages](#)

KM: [BAM BAM Task](#)
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<th>Proportional reasoning</th>
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<tr>
<td><strong>Key concepts</strong></td>
</tr>
<tr>
<td>• express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)</td>
</tr>
<tr>
<td>• identify and work with fractions in ratio problems</td>
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<tr>
<td>• understand and use proportion as equality of ratios</td>
</tr>
<tr>
<td>• express a multiplicative relationship between two quantities as a ratio or a fraction</td>
</tr>
<tr>
<td>• use compound units (such as speed, rates of pay, unit pricing)</td>
</tr>
<tr>
<td>• change freely between compound units (e.g. speed, rates of pay, prices) in numerical contexts</td>
</tr>
<tr>
<td>• relate ratios to fractions and to linear functions</td>
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<tr>
<td><strong>The Big Picture:</strong> Ratio and Proportion progression map</td>
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<table>
<thead>
<tr>
<th><strong>Possible learning intentions</strong></th>
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<tbody>
<tr>
<td>• Explore the uses of ratio</td>
</tr>
<tr>
<td>• Investigate the connection between ratio and proportion</td>
</tr>
<tr>
<td>• Solve problems involving proportional reasoning</td>
</tr>
<tr>
<td>• Solve problems involving compound units</td>
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<table>
<thead>
<tr>
<th><strong>Possible success criteria</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Identify ratio in a real-life context</td>
</tr>
<tr>
<td>• Write a ratio to describe a situation</td>
</tr>
<tr>
<td>• Identify proportion in a situation</td>
</tr>
<tr>
<td>• Find a relevant multiplier in a situation involving proportion</td>
</tr>
<tr>
<td>• Use fractions fluently in situations involving ratio or proportion</td>
</tr>
<tr>
<td>• Understand the connections between ratios and fractions</td>
</tr>
<tr>
<td>• Understand the meaning of a compound unit</td>
</tr>
<tr>
<td>• Know the connection between speed, distance and time</td>
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<tr>
<td>• Solve problems involving speed</td>
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<tr>
<td>• Identify when it is necessary to convert quantities in order to use a sensible unit of measure</td>
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<table>
<thead>
<tr>
<th><strong>Prerequisites</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Understand and use ratio notation</td>
</tr>
<tr>
<td>• Divide an amount in a given ratio</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Mathematical language</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratio</td>
</tr>
<tr>
<td>Proportion</td>
</tr>
<tr>
<td>Proportional</td>
</tr>
<tr>
<td>Multiplier</td>
</tr>
<tr>
<td>Speed</td>
</tr>
<tr>
<td>Unitary method</td>
</tr>
<tr>
<td>Units</td>
</tr>
<tr>
<td>Compound unit</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Notation</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Kilometres per hour is written as km/h or kmh⁻¹</td>
</tr>
<tr>
<td>Metres per second is written as m/s or ms⁻¹</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Pedagogical notes</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The Bar Model is a powerful strategy for pupils to ‘re-present’ a problem involving ratio.</td>
</tr>
<tr>
<td>NCETM: The Bar Model</td>
</tr>
<tr>
<td>NCETM: Multiplicative reasoning</td>
</tr>
<tr>
<td>NCETM: Departmental workshops: Proportional Reasoning</td>
</tr>
<tr>
<td>NCETM: Glossary</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Reasoning opportunities and probing questions</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Show me an example of two quantities that will be in proportion. And another. And another ...</td>
</tr>
<tr>
<td>• (Showing a table of values such as the one below) convince me that this information shows a proportional relationship</td>
</tr>
<tr>
<td>6   9</td>
</tr>
<tr>
<td>10  15</td>
</tr>
<tr>
<td>14  21</td>
</tr>
<tr>
<td>• Which is the faster speed: 60 km/h or 10 m/s? Explain why.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Suggested activities</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>KM: Proportion for real</td>
</tr>
<tr>
<td>KM: Investigating proportionality</td>
</tr>
<tr>
<td>KM: Maths to infinity: Fractions, decimals, percentages, ratio, proportion</td>
</tr>
<tr>
<td>NRICH: In proportion</td>
</tr>
<tr>
<td>NRICH: Ratio or proportion?</td>
</tr>
<tr>
<td>NRICH: Roasting old chestnuts 3</td>
</tr>
<tr>
<td>Standards Unit: N6 Developing proportional reasoning</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Possible misconceptions</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts</td>
</tr>
<tr>
<td>• Some pupils may think that a multiplier always has to be greater than 1</td>
</tr>
<tr>
<td>• When converting between times and units, some pupils may base their working on 100 minutes = 1 hour</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Learning review</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>KM: BAM Task</td>
</tr>
</tbody>
</table>
**Pattern sniffing**

**Key concepts**
- generate terms of a sequence from either a term-to-term or a position-to-term rule
- deduce expressions to calculate the nth term of linear sequences

**The Big Picture:** Algebra progression map

**Possible learning intentions**
- Explore sequences
- Generate a sequence from a term-to-term rule
- Understand the meaning of a position-to-term rule
- Use a position-to-term rule to generate a sequence
- Find the position-to-term rule for a given sequence
- Use algebra to describe the position-to-term rule of a linear sequence (the nth term)
- Use the nth term of a sequence to deduce if a given number is in a sequence
- Generate a sequence using a spreadsheet

**Possible success criteria**
- Generate a sequence from a term-to-term rule
- Understand the meaning of a position-to-term rule
- Use a position-to-term rule to generate a sequence
- Find the position-to-term rule for a given sequence
- Use algebra to describe the position-to-term rule of a linear sequence (the nth term)
- Use the nth term of a sequence to deduce if a given number is in a sequence
- Generate a sequence using a spreadsheet

**Prerequisites**
- Use a term-to-term rule to generate a sequence
- Find the term-to-term rule for a sequence
- Describe a sequence using the term-to-term rule

**Mathematical language**
- Sequence
- Linear
- Term
- Difference
- Term-to-term rule
- Position-to-term rule
- Ascending
- Descending

**Notation**
- T(n) is often used when finding the nth term of sequence

**Pedagogical notes**
- Using the nth term for times tables is a powerful way of finding the nth term for any linear sequence. For example, if the pupils understand the 3 times table can be described as ‘3n’ then the linear sequence 4, 7, 10, 13, … can be described as the 3 times table ‘shifted up’ one place, hence 3n + 1.
- Exploring statements such as ‘is 171 is in the sequence 3, 9, 15, 21, 27, ..?’ is a very powerful way for pupils to realise that ‘term-to-term’ rules can be inefficient and therefore ‘position-to-term’ rules (nth term) are needed.

**Reasoning opportunities and probing questions**
- Show me a sequence that could be generated using the nth term 4n ± c. And another. And another ...
- What’s the same, what’s different: 4, 7, 10, 13, 16, .... , 2, 5, 8, 11, 14, .... , 4, 9, 14, 19, 24, .... and 4, 10, 16, 22, 28, ....?
- The 4th term of a linear sequence is 15. Show me the nth term of a sequence with this property. And another. And another ...
- Convince me that the nth term of the sequence 2, 5, 8, 11, ... is 3n - 1.
- Kenny says the 171 is in the sequence 3, 9, 15, 21, 27, ... Do you agree with Kenny? Explain your reasoning.

**Suggested activities**
- KM: Spreadsheet sequences
- KM: Generating sequences
- KM: Maths to Infinity: Sequences
- KM: Stick on the Maths: Linear sequences
- NRICH: Charlie’s delightful machine
- NRICH: A little light thinking
- NRICH: Go forth and generalise

**Learning review**
- KM: 8M9 BAM Task

**Possible misconceptions**
- Some pupils will think that the nth term of the sequence 2, 5, 8, 11, ... is n + 3.
- Some pupils may think that the (2n)th term is double the nth term of a linear sequence.
- Some pupils may think that sequences with nth term of the form ‘ax ± b’ must start with ‘a’.
### Investigating angles

**Key concepts**
- understand and use alternate and corresponding angles on parallel lines
- derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

**The Big Picture:** Position and direction progression map

### Possible learning intentions
- Develop knowledge of angles
- Explore geometrical situations involving parallel lines

### Possible success criteria
- Identify alternate angles and know that they are equal
- Identify corresponding angles and know that they are equal
- Use knowledge of alternate and corresponding angles to calculate missing angles in geometrical diagrams
- Establish the fact that angles in a triangle must total 180°
- Use the fact that angles in a triangle total 180° to work out the total of the angles in any polygon
- Establish the size of an interior angle in a regular polygon
- Know the total of the exterior angles in any polygon
- Establish the size of an exterior angle in a regular polygon

### Prerequisites
- Use angles at a point, angles at a point on a line and vertically opposite angles to calculate missing angles in geometrical diagrams
- Know that the angles in a triangle total 180°

### Mathematical language
- **Degrees**
- Right angle, acute angle, obtuse angle, reflex angle
- Vertically opposite
- Geometry, geometrical
- Parallel
- Alternate angles, corresponding angles
- Interior angle, exterior angle
- Regular polygon

**Notation**
- Dash notation to represent equal lengths in shapes and geometric diagrams
- Arrow notation to show parallel lines

### Pedagogical notes
- The KM: Perplexing parallels resource is a great way for pupils to discover practically the facts for alternate and corresponding angles.
- Pupils have established the fact that angles in a triangle total 180° in Stage 7. However, using alternate angles they are now able to prove this fact.
- Encourage pupils to draw regular and irregular convex polygons to discover the sum of the interior angles = \((n - 2) \times 180°\).
- NCETM: Glossary

### Common approaches
- Teachers insist on correct mathematical language (and not F-angles or Z-angles for example)

### Reasoning opportunities and probing questions
- Show me a pair of alternate (corresponding) angles. And another. And another …
- Jenny thinks that hexagons are the only polygon that tessellates. Do you agree? Explain your reasoning.
- Convince me that the angles in a triangle total 180°.
- Convince me that the interior angle of a pentagon is 540°.
- Always/Sometimes/Never: The sum of the interior angles of an \(n\)-sided polygon can be calculated using \(\text{Sum} = (n - 2) \times 180°\).
- Always/Sometimes/Never: The sum of the exterior angles of a polygon is 360°.

### Suggested activities
- KM: Alternate and corresponding angles
- KM: Perplexing parallels
- KM: Investigating polygons
- KM: Maths to infinity: Lines and angles
- KM: Stick on the Maths: Alternate and corresponding angles
- KM: Stick on the Maths: Geometrical problems
- NRICH: Ratty

### Learning review
- www.diagnosticquestions.com

### Possible misconceptions
- Some pupils may think that alternate and/or corresponding angles have a total of 180° rather than being equal.
- Some pupils may think that the sum of the interior angles of an \(n\)-sided polygon can be calculated using \(\text{Sum} = n \times 180°\).
- Some pupils may think that the sum of the exterior angles increases as the number of sides of the polygon increases.
### Calculating fractions, decimals and percentages

**6 hours**

**Key concepts**
- interpret fractions and percentages as operators
- work with percentages greater than 100%
- solve problems involving percentage change, including original value problems, and simple interest including in financial mathematics
- calculate exactly with fractions

**Possible learning intentions**
- Calculate with fractions
- Calculate with percentages

**Possible success criteria**
- Recognise when a fraction (percentage) should be interpreted as a number
- Recognise when a fraction (percentage) should be interpreted as an operator
- Identify the multiplier for a percentage increase or decrease when the percentage is greater than 100%
- Use calculators to increase an amount by a percentage greater than 100%
- Solve problems involving percentage change
- Solve original value problems when working with percentages
- Solve financial problems including simple interest
- Understand the meaning of giving an exact solution
- Solve problems that require exact calculation with fractions

**Prerequisites**
- Apply the four operations to proper fractions, improper fractions and mixed numbers
- Use calculators to find a percentage of an amount using multiplicative methods
- Identify the multiplier for a percentage increase or decrease
- Use calculators to increase (decrease) an amount by a percentage using multiplicative methods
- Know that percentage change = actual change ÷ original amount

**Mathematical language**
- Proper fraction, improper fraction, mixed number
- Simplify, cancel, lowest terms
- Percent, percentage
- Percentage change
- Original amount
- Multiplier
- (Simple) interest
- Exact

**Pedagogical notes**
- The bar model is a powerful strategy for pupils to ‘re-present’ a problem involving percentage change.
- Only simple interest should be explored in this unit. Compound interest will be developed later.

**Common approaches**
- When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy
- Teachers use the horizontal fraction bar notation at all times

**Reasoning opportunities and probing questions**
- Convince me that the multiplier for a 150% increase is 2.5
- Kenny buys a poncho in a 25% sale. The sale price is £40. Kenny thinks that the original is £50. Do you agree with Kenny? Explain your answer.
- Jenny thinks that increasing an amount by 200% is the same as multiplying by 3. Do you agree with Jenny? Explain your answer.

**Suggested activities**
- KM: Stick on the Maths: Proportional reasoning
- KM: Stick on the Maths: Multiplicative methods
- KM: Percentage identifying
- NRICH: One or both
- NRICH: Antiques roadshow

**Possible misconceptions**
- Some pupils may think that the multiplier for a 150% increase is 1.5
- Some pupils may think that increasing an amount by 200% is the same as doubling.
- In isolation, pupils may be able to solve original value problems confidently. However, when it is necessary to identify the type of percentage problem, many pupils will apply a method for a more simple percentage increase / decrease problem. If pupils use models (e.g. the bar model, or proportion tables) to represent all problems then they are less likely to make errors in identifying the type of problem.
## Solving equations and inequalities

### Key concepts
- solve linear equations with the unknown on both sides of the equation
- find approximate solutions to linear equations using a graph

### Possible learning intentions
- Solve linear equations with the unknown on one side
- Solve linear equations with the unknown on both sides
- Explore connections between graphs and equations

### Possible success criteria
- Identify the correct order of undoing the operations in an equation
- Solve linear equations with the unknown on one side when the solution is a negative number
- Solve linear equations with the unknown on both sides when the solution is a whole number
- Solve linear equations with the unknown on both sides when the solution is a fraction
- Solve linear equations with the unknown on both sides when the solution involves brackets
- Recognise that the point of intersection of two graphs corresponds to the solution of a connected equation
- Check the solution to an equation by substitution

**Prerequisites**
- Choose the required inverse operation when solving an equation
- Solve linear equations by balancing when the solution is a whole number or a fraction

**Mathematical language**
- Algebra, algebraic, algebraically
- Unknown
- Equation
- Operation
- Solve
- Solution
- Brackets
- Symbol
- Substitute
- Graph
- Point of intersection

**Notation**
The lower case and upper case of a letter should not be used interchangeably when worked with algebra. Juxtaposition is used in place of ‘×’. 2a is used rather than a2. Division is written as a fraction.

**Reasoning opportunities and probing questions**
- Show me an (one-step, two-step) equation with a solution of -8 (negative, fractional solution). And another. And another ...
- What’s the same, what’s different: 2x + 7 = 25, 3x + 7 = x + 25, x + 7 = 7 – x, 4x + 14 = 50?
- Convince me how you could use graphs to find solutions, or estimates, for equations.

**Suggested activities**
- KM: Solving equations
- KM: Stick on the Maths: Constructing and solving equations
- NRICH: Think of Two Numbers
- Learning review
- KM: BM10 BAM Task

**Possible misconceptions**
- Some pupils may think that you always have to manipulate the equation to have the unknowns on the LHS of the equal sign, for example 2x = 3 = 6x + 6
- Some pupils think if 4x = 2 then x = 2.
- When solving equations of the form 2x – 8 = 4 – x, some pupils may subtract ‘x’ from both sides.
### Calculating Space

#### Key concepts
- compare lengths, areas and volumes using ratio notation
- calculate perimeters of 2D shapes, including circles
- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference
- know the formulae: circumference of a circle = 2πr, area of a circle = πr²
- calculate areas of circles and composite shapes
- know and apply formulae to calculate volume of right prisms (including cylinders)

#### Possible learning intentions
- Investigate circles
- Discover π
- Solve problems involving circles
- Explore prisms and cylinders

#### Prerequisites
- Know how to use formulae to find the area of rectangles, parallelograms, triangles and trapezia
- Know how to find the area of compound shapes

#### Mathematical language
- Circle
- Centre
- Radius, diameter, chord, circumference
- Pi
- (Right) prism
- Cross-section
- Cylinder
- Polygon, polygonal
- Solid

#### Notation
- π

#### Abbreviations of units in the metric system: km, m, cm, mm

#### Possible success criteria
- Know the vocabulary of circles
- Know that the number π (pi) = 3.1415926535...
- Recall π to two decimal places
- Know the formula circumference of a circle = 2πr = πd
- Calculate the circumference of a circle when radius (diameter) is given
- Calculate the radius (diameter) of a circle when the circumference is known
- Calculate the perimeter of composite shapes that include sections of a circle
- Know the formula area of a circle = πr²
- Calculate the area of a circle when radius (diameter) is given
- Calculate the radius (diameter) of a circle when the area is known
- Calculate the area of composite shapes that include sections of a circle
- Know the formula for finding the volume of a right prism (cylinder)
- Calculate the volume of a right prism (cylinder)

#### Reasoning opportunities and probing questions
- Convince me C = 2πr = πd.
- What is wrong with this statement? How can you correct it?
  The area of a circle with radius 7 cm is approximately 441 cm² because (3 × 7²) = 441.
- Convince me the area of a semi-circle = \( \frac{\pi d^2}{4} \).
- Name a right prism. And another. And another ...
- Convince me that a cylinder is not a prism

#### Suggested activities
- KM: Circle connections, Circle connections v2
- KM: Circle circumferences, Circle problems
- KM: Maths to Infinity: Area and Volume
- KM: Stick on the Maths: Circumference and area of a circle
- KM: Stick on the Maths: Right prisms
- NRICH: Blue and White
- NRICH: Efficient Cutting
- NRICH: Cola Can
- Learning review
- KM: 8M12 BAM Task

#### Possible misconceptions
- Some pupils will work out (π × radius)² when finding the area of a circle
- Some pupils may use the sloping height when finding cross-sectional areas that are parallelograms, triangles or trapezia
- Some pupils may think that the area of a triangle = base × height
- Some pupils may think that you multiply all the numbers to find the volume of a prism
- Some pupils may confuse the concepts of surface area and volume

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**The Big Picture:** Measurement and mensuration progression map

**NRICH:**
- Cola Can
- Efficient Cutting
- Math to Infinity
- Stick on the Maths: Right prisms
- Stick on the Maths: Circumference and area of a circle
- Blue and White

**KM:**
- Circle connections
- Circle connections v2
- Circle circumferences
- Maths to Infinity: Area and Volume
- Stick on the Maths: Right prisms
- Stick on the Maths: Circumference and area of a circle

**Abbreviations of units in the metric system:** km, m, cm, mm, mm², m², km², mm³, cm³, km³

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**NRICH:**
- Cola Can
- Efficient Cutting
- Math to Infinity
- Stick on the Maths: Right prisms
- Stick on the Maths: Circumference and area of a circle
- Blue and White

**KM:**
- Circle connections
- Circle connections v2
- Circle circumferences
- Maths to Infinity: Area and Volume
- Stick on the Maths: Right prisms
- Stick on the Maths: Circumference and area of a circle

**Abbreviations of units in the metric system:** km, m, cm, mm², m², km², mm³, cm³, km³
### Algebraic proficiency: visualising

#### Key concepts
- plot graphs of equations that correspond to straight-line graphs in the coordinate plane
- identify and interpret gradients and intercepts of linear functions graphically
- recognise, sketch and interpret graphs of linear functions and simple quadratic functions
- plot and interpret graphs and graphs of non-standard (piece-wise linear) functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance and speed

#### The Big Picture: Algebra progression map

#### Possible learning intentions
- Plot and interpret linear graphs
- Plot and quadratic graphs
- Model real situations using linear graphs

#### Possible success criteria
- Know that graphs of functions of the form \( y = mx + c \), \( x \pm y = c \) and \( ax \pm by = c \) are linear
- Plot graphs of functions of the form \( y = mx + c \) \((x \pm y = c, ax \pm by = c)\)
- Understand the concept of the gradient of a straight line
- Find the gradient of a straight line on a unit grid
- Find the \( y \)-intercept of a straight line
- Sketch a linear graph
- Distinguish between a linear and quadratic graph
- Plot graphs of quadratic functions of the form \( y = ax^2 \pm c \)
- Sketch a simple quadratic graph
- Plot and interpret graphs of piece-wise linear functions in real contexts
- Plot and interpret distance-time graphs (speed-time graphs)
- Find approximate solutions to kinematic problems involving distance and speed

#### Prerequisites
- Use coordinates in all four quadrants
- Write the equation of a line parallel to the \( x \)-axis or the \( y \)-axis
- Draw a line parallel to the \( x \)-axis or the \( y \)-axis given its equation
- Identify the lines \( y = x \) and \( y = -x \)
- Draw the lines \( y = x \) and \( y = -x \)
- Substitute positive and negative numbers into formulae

#### Mathematical language

<table>
<thead>
<tr>
<th>Plot</th>
<th>Equation (of a graph)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function</td>
<td>Formula</td>
</tr>
<tr>
<td>Linear</td>
<td>Coordinate plane</td>
</tr>
<tr>
<td>Gradient</td>
<td>( y )-intercept</td>
</tr>
<tr>
<td>Substitute</td>
<td>Quadratic</td>
</tr>
<tr>
<td>Piece-wise linear</td>
<td>Model</td>
</tr>
<tr>
<td>Kinematic, Speed, Distance</td>
<td></td>
</tr>
</tbody>
</table>

#### Notation

| \( y = mx + c \) |

#### Pedagogical notes

- When plotting graphs of functions of the form \( y = mx + c \) a table of values can be useful. Note that negative number inputs can cause difficulties. Pupils should be aware that the values they have found for linear functions should correspond to a straight line.
- NCETM: [Glossary](#)
- **Common approaches**
  - Pupils are taught to use positive numbers wherever possible to reduce potential difficulties with substitution of negative numbers
  - Students plot points with a ‘\( x \)’ and not ‘\( -x \)’
  - Students draw graphs in pencil
  - All pupils use dynamic geometry software to explore graphs of functions

#### Reasoning opportunities and probing questions

- Draw a distance-time graph of your journey to school. Explain the key features.
- Show me a point on this line (e.g. \( y = 2x + 1 \)). And another, and another …
- (Given an appropriate distance-time graph) convince me that Kenny is stationary between 10:00 a.m. and 10:45 a.m.

#### Suggested activities

- KM: [Matching graphs](#)
- KM: [Autograph 1](#)
- KM: [Autograph 2](#)
- KM: [The hare and the tortoise](#)
- KM: [8M11 BAM Task](#)

#### Possible misconceptions

- When plotting linear graphs some pupils may draw a line segment that stops at the two most extreme points plotted
- Some pupils may think that a sketch is a very rough drawing. It should still identify key features, and look neat, but will not be drawn to scale
- Some pupils may think that a positive gradient on a distance-time graph corresponds to a section of the journey that is uphill
- Some pupils may think that the graph \( y = x^2 + c \) is the graph of \( y = x^2 \) translated horizontally.
### Understanding risk II

#### Key concepts
- apply systematic listing strategies
- record describe and analyse the frequency of outcomes of probability experiments using frequency trees
- enumerate sets and combinations of sets systematically, using tables, grids and Venn diagrams
- construct theoretical possibility spaces for combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
- apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments

#### The Big Picture: Probability progression map

#### Possible learning intentions
- Explore experiments and outcomes
- Develop understanding of probability
- Use probability to make predictions

#### Possible success criteria
- List all elements in a combination of sets using a Venn diagram
- List outcomes of an event systematically
- Use a table to list all outcomes of an event
- Use frequency trees to record outcomes of probability experiments
- Make conclusions about probabilities based on frequency trees
- Construct theoretical possibility spaces for combined experiments with equally likely outcomes
- Calculate probabilities using a possibility space
- Use theoretical probability to calculate expected outcomes
- Use experimental probability to calculate expected outcomes

#### Prerequisites
- Convert between fractions, decimals and percentages
- Understand the use of the 0-1 scale to measure probability
- Work out theoretical probabilities for events with equally likely outcomes
- Know how to represent a probability
- Know that the sum of probabilities for all outcomes is 1

#### Mathematical language

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency tree</td>
<td></td>
</tr>
<tr>
<td>Enumerate</td>
<td></td>
</tr>
<tr>
<td>Set</td>
<td></td>
</tr>
<tr>
<td>Venn diagram</td>
<td></td>
</tr>
<tr>
<td>Possibility space, sample space</td>
<td></td>
</tr>
<tr>
<td>Equally likely outcomes</td>
<td></td>
</tr>
<tr>
<td>Theoretical probability</td>
<td></td>
</tr>
<tr>
<td>Random</td>
<td></td>
</tr>
<tr>
<td>Bias, Fairness</td>
<td></td>
</tr>
<tr>
<td>Relative frequency</td>
<td></td>
</tr>
</tbody>
</table>

#### Possible misconceptions
- Some students may think that there are only three outcomes when two coins are flipped, or that there are only six outcomes when three coins are flipped
- Some students may think that there are 12 unique outcomes when two dice are rolled
- Some students may think that there are 12 possible totals when two dice are rolled

#### Learning review
- www.diagnosticquestions.com

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**Note:** The Venn diagram was invented by John Venn (1834 – 1923)
### Presentation of data

#### Key concepts
- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate graphical representation involving discrete, continuous and grouped data
- use and interpret scatter graphs of bivariate data
- recognise correlation

#### Possible learning intentions
- Explore types of data
- Construct and interpret graphs
- Select appropriate graphs and charts

#### Possible success criteria
- Know the meaning of continuous data
- Interpret a grouped frequency table for continuous data
- Construct a grouped frequency table for continuous data
- Construct histograms for grouped data with equal class intervals
- Interpret histograms for grouped data with equal class intervals
- Construct and use the horizontal axis of a histogram correctly
- Plot a scatter diagram of bivariate data
- Understand the meaning of ‘correlation’
- Interpret a scatter diagram using understanding of correlation

#### Prerequisites
- Mathematical language
  - Data
  - Categorical data, Discrete data
  - Continuous data, Grouped data
  - Table, Frequency table
  - Frequency
  - Histogram
  - Scale, Graph
  - Axis, axes
  - Scatter graph (scatter diagram, scattergram, scatter plot)
  - Bivariate data
  - (Linear) Correlation
  - Positive correlation, Negative correlation

#### Mathematical language
- Notation
  - Correct use of inequality symbols when labeling groups in a frequency table

#### Pedagogical notes
- The word histogram is often misused and an internet search of the word will usually reveal a majority of non-histograms. The correct definition is ‘a diagram made of rectangles whose areas are proportional to the frequency of the group’. If the class widths are equal, as they are in this unit of work, then the vertical axis shows the frequency. It is only later that pupils need to be introduced to unequal class widths and frequency density.
- Lines of best fit on scatter diagrams are not introduced until Stage 9, although pupils may well have encountered both lines and curves of best fit in science by this time.

#### Reasoning opportunities and probing questions
- Show me a scatter graph with positive (negative, no) correlation. And another. And another. And another. And another. And another. And another. And another. And another. And another. And another. And another. And another. And another. And another. And another. And another. And another. Kenny thinks that histogram is just a ‘fancy’ name for a bar chart. Do you agree with Kenny? Explain your answer.
- What’s the same and what’s different: histogram, scatter diagram, bar chart, pie chart?
- Always/Sometimes/Never: A scatter graph

#### Suggested activities
- KM: Make a ‘human’ scatter graph by asking pupils to stand at different points on a giant set of axes.
- KM: Spreadsheet statistics
- KM: Stick on the Maths HD2: Selecting and constructing graphs and charts
- KM: Stick on the Maths HD3: Working with grouped data
- Learning review [www.diagnosticquestions.com](http://www.diagnosticquestions.com)

#### Possible misconceptions
- Some pupils may label the bar of a histogram rather than the boundaries of the bars
- Some pupils may leave gaps between the bars in a histogram
- Some pupils may misuse the inequality symbols when working with a grouped frequency table
### Measuring data

**Key concepts**
- Interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers).
- Apply statistics to describe a population.

**Possible learning intentions**
- Investigate averages
- Explore ways of summarising data
- Analyse and compare sets of data

**Possible success criteria**
- Find the modal class of set of grouped data
- Find the class containing the median of a set of data
- Find the midpoint of a class
- Calculate an estimate of the mean from a grouped frequency table
- Estimate the range from a grouped frequency table
- Analyse and compare sets of data
- Appreciate the limitations of different statistics (mean, median, mode, range)
- Choose appropriate statistics to describe a set of data
- Justify choice of statistics to describe a set of data

**Prerequisites**
- Understand the mean, mode and median as measures of typicality (or location)
- Find the mean, median, mode and range of a set of data
- Find the mean, median, mode and range from a frequency table

**Mathematical language**
- Average
- Spread
- Consistency
- Mean
- Median
- Mode
- Range
- Statistic
- Statistics
- Approximate, Round
- Calculate an estimate
- Grouped frequency
- Midpoint

**Notation**
- Correct use of inequality symbols when labeling groups in a frequency table

**Pedagogical notes**
- The word ‘average’ is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the ‘arithmetic mean’).
- NCETM: Glossary

**Common approaches**
- Every classroom has a set of statistics posters on the wall.
- All students are taught to use mathematical presentation correctly when calculating and rounding solutions, e.g. \((21 + 56 + 35 + 12) ÷ 30 = 124 ÷ 30 = 41.3\) to 1 d.p.

**Reasoning opportunities and probing questions**
- Show me an example of an outlier. And another. And another.
- Convince me why the mean from a grouped set of data is only an estimate.
- What’s the same and what’s different: mean, modal class, median, range?
- Always/Sometimes/Never: A set of grouped data will have one modal class.
- Convince me how to estimate the range for grouped data.

**Suggested activities**
- KM: Swillions
- KM: Lottery project
- NRICH: Half a Minute
- Learning review [www.diagnosticquestions.com](http://www.diagnosticquestions.com)

**Possible misconceptions**
- Some pupils may incorrectly estimate the mean by dividing the total by the numbers of groups rather than the total frequency.
- Some pupils may incorrectly think that there can only be one modal class.
- Some pupils may incorrectly estimate the range of grouped data by subtracting the upper bound of the first group from the lower bound of the last group.