### Mathematics overview: Stage 6

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### Numbers and the number system

#### Key concepts
- Identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places.
- Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit.
- Use negative numbers in context, and calculate intervals across zero.
- Identify common factors, common multiples and prime numbers.

#### Possible learning intentions
- Understand and use decimals with up to three decimal places.
- Work with numbers up to ten million.
- Explore the use of negative numbers.
- Investigate prime numbers.

#### Possible success criteria
- Understand place value in numbers with up to three decimal places.
- Multiply whole numbers by 10 (100, 1000).
- Divide whole numbers by 10 (100, 1000) when the answer is a whole number.
- Multiply (divide) numbers with up to three decimal places by 10 (100, 1000).
- Understand (order, write, read) place value in numbers with up to eight digits.
- Understand and use negative numbers when working with other contexts.
- Know the meaning of a common multiple (factor) of two numbers.
- Identify common multiples (factors) of two numbers.
- Know how to test if a number up to 120 is prime.

#### Prerequisites
- Understand and use place value in numbers with up to seven digits.
- Multiply and divide whole numbers by 10, 100, 1000.
- Multiply and divide numbers with one decimal place by 10, 100, 1000.
- Know the meaning of ‘factor’ and ‘multiple’ and ‘prime’.

#### Mathematical language
- Place value
- Digit
- Negative number
- (Common) multiple
- (Common) factor
- Divisible
- Prime number, Composite number

#### Pedagogical notes
- Zero is neither positive nor negative.
- When multiplying and dividing by powers of ten, the decimal point is fixed and it is the digits that move.
- Ensure that pupils can deal with large numbers that include zeros in the HTh and/or H column (e.g., 43,006,619).

#### Common approaches
- The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.

### Reasoning opportunities and probing questions

- Convince me that 109 is a prime number.
- Look at this number (24 054 028). Show me another number (with 4, 5, 6, 7 digits) that includes a 5 with the same value. And another. And another....

#### Suggested activities
- KM: Maths to Infinity: Directed numbers
- KM: Extend the idea of Eratosthenes' sieve to a 12 by 12 grid
- KM: Exploring primes activities: Artistic Eratosthenes sieve
- KM: Use Powers of ten to demonstrate connections.
- NRICH: Factor-multiple chains
- NRICH: The Moons of Vuvv
- NRICH: Round and round the circle
- NRICH: Counting cogs
- Learning review
  - www.diagnosticquestions.com

#### Possible misconceptions
- Some pupils can confuse the language of large (and small) numbers since the prefix ‘milli-’ means ‘one thousandth’ (meaning that there are 1000 millimetres in a metre for example) while one million is actually a thousand thousand.
- Some pupils may not realise that degrees (°) and degrees Celsius (°C) are two different and distinct units of measurement.
- Some pupils may think that 1 is a prime number.
### Calculating

#### Key concepts
- Perform mental calculations, including with mixed operations and large numbers
- Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- Solve problems involving addition, subtraction and multiplication
- Use their knowledge of the order of operations to carry out calculations

#### Possible learning intentions
- Develop mental calculation skills
- Extend written methods of multiplication
- Know and use the order of operations
- Solve problems involving addition, subtraction and multiplication

**Bring on the Maths**: Moving on up!
Calculating: #4
Fractions, decimals & percentages: #6
Solving problems: #2

#### Prerequisites
- Recall multiplication facts for multiplication tables up to 12 × 12
- Recall division facts for multiplication tables up to 12 × 12
- Understand the commutativity of multiplication and addition
- Multiply a three-digit number by a two-digit number using short multiplication
- Use column addition and subtraction for numbers with more than four digits

**Bring on the Maths**: Moving on up!
Calculating: #1
Solving problems: #1

#### Possible success criteria
- Combine addition and subtraction when multiplying mentally
- Multiply a two-digit number by a single-digit number mentally
- Add a three-digit number to a two-digit number mentally (when bridging of hundreds is required)
- Multiply a four-digit number by a two-digit number using long multiplication
- Identify when addition, subtraction or multiplication is needed as part of solving multi-step problems
- Explain why addition or subtraction is needed at any point when solving multi-step problems
- Solve multi-step problems involving addition, subtraction and/or multiplication
- Know that addition and subtraction have equal priority
- Know that multiplication and division have equal priority
- Know that multiplication and division take priority over addition and subtraction

#### Mathematical language
- Addition
- Subtraction
- Sum, Total
- Difference, Minus, Less
- Column addition
- Column subtraction
- Operation
- Multiply, Multiplication, Times, Product
- Commutative
- Factor
- Short multiplication
- Long multiplication
- Estimate

#### Pedagogical notes
Note that if not understood fully, BIDMAS can give the wrong answer to a calculation; e.g. 6 – 2 ÷ 3.
The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements.
Use a basic and scientific calculator to work out 2 + 3.

#### Possible misconceptions
- Some pupils may write statements such as 140 – 190 = 50.
- When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166.
- The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations.

### Reasoning opportunities and probing questions
- Find missing digits in otherwise completed long multiplication calculations
- Convince me that 2472 × 12 = 29664
- Why have you chosen to add (subtract, multiply)?

**NCETM**: Addition and Subtraction Reasoning
**NCETM**: Multiplication and Division Reasoning

#### Suggested activities
- **KM**: Long multiplication template
- **KM**: Maximise, minimise. Adapt ideas to fit learning intentions.
- **KM**: Maths to Infinity: Complements
- **KM**: Maths to Infinity: Multiplying and dividing
- **NRICH**: Become Maths detectives
- **NRICH**: Exploring number patterns you make
- **NRICH**: Reach 100
- Learning review
  - [www.diagnosticquestions.com](http://www.diagnosticquestions.com)

### Possible misconceptions
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- The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations.
Calculating: division

Key concepts
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division; interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- use written division methods in cases where the answer has up to two decimal places
- solve problems involving division
- use their knowledge of the order of operations to carry out calculations involving the four operations

Possible learning intentions
- Develop written methods of division
- Deal with remainders when carrying out division
- Solve problems involving the four operations

Prerequisites
- Use knowledge of multiplication tables when dividing
- Know how to use short division

Possible success criteria
- Use short division to divide a four-digit number by a one-digit number
- Use short division to divide a three- (or four-) digit number by a two-digit number
- Understand the method of long division
- Use long division to find the remainder at each step of the division
- Know how to write, and use, the remainder at each step of the division
- Use long division to divide a three- (or four-) digit number by a two-digit number
- Write the remainder to a division problem as a remainder
- Write the remainder to a division problem as a fraction
- Extend beyond the decimal point to write the remainder as a decimal
- Identify when division is needed to solve a problem
- Extract the correct information from a problem and set up a written division calculation
- Interpret a remainder when carrying out division

Possible misconceptions
- Some pupils may write statements such as $12 \div 132 = 11$
- Formal written methods of addition, subtraction and multiplication work from right to left. Formal division works from left to right.
- When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example:

```
  0  10  7  5
8  3  4  6  9
```
3 $\div$ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been ‘moved across’ and therefore everything that follows has been correctly carried out based on an early misunderstanding.

Return to overview
### Visualising and constructing

**8 hours**

#### Key concepts
- draw 2-D shapes using given dimensions and angles
- recognise, describe and build simple 3-D shapes, including making nets

#### Possible learning intentions
- Construct 2D shapes
- Investigate 3D shapes
- Explore nets of 3D shapes

#### Prerequisites
- Know the names of common 2D shapes
- Know the names of common 3D shapes
- Use a protractor to measure and draw angles

#### Mathematical language
- Protractor
- Measure
- Nearest
- Construct
- Sketch
- Cube, Cuboid, Cylinder, Pyramid, Prism
- Net
- Edge, Face, Vertex (Vertices)
- Visualise

#### Notation
- Dash notation to represent equal lengths in shapes and geometric diagrams
- Right angle notation

#### Possible success criteria
- Use a protractor to draw angles up to 180°
- Use a protractor to work out and construct reflex angles
- Use a ruler to draw lines to the nearest millimetre
- Use squared paper to guide construction of 2D shapes
- Complete tessellations of given shapes
- Know the names of common 3D shapes
- Use mathematical language to describe 3D shapes
- Construct 3D shapes from given nets
- Use ‘Polydron’ to construct nets for common 3D shapes
- Draw accurate nets for common 3D shapes
- Find all the nets for a cube
- Use a net to visualise the edges (vertices) that will meet when folded

#### Prerequisites
- Mathematical language
- Pedagogical notes

- A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid.
- A cube is a special case of a cuboid, and a cuboid is a special case of a prism.
- Many pupils struggle to sketch 3D shapes. A good strategy for any type of prism is to draw the cross-section (using squares for guidance), and then draw a second identical shape offset from the first. The matching corners can then be joined with straight lines. Some dotted lines (or rubbing out of lines) will be required.

#### Reasoning opportunities and probing questions
- Show me an example of a net of a cube. And another. And another.
- What is wrong with this attempt at a net of a cuboid? How can it be changed?
- How many different ways are there to complete these nets?

#### Suggested activities
- KM: Visualising 3D shapes
- KM: Tessellating Tess
- KM: Fibonacci’s disappearing squares
- KM: Unravelling dice
- NRICH: Making spirals
- NRICH: Cut nets
- NRICH: Making cuboids

#### Learning review
- www.diagnosticquestions.com

#### Possible misconceptions
- Some pupils will read the wrong way round the scale on a typical semi-circular protractor, therefore using 180° - required angle
- Some pupils may measure from the end of a ruler, rather than the start of the measuring scale
- Some pupils may think that several repeats of a shape in any pattern constitutes a tessellation
- When given a net of a 3D shape some pupils may think that the number of vertices of the 3D shape is found by counting the number of ‘corners’ on the net

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**NCETM:** Geometry - Properties of Shapes Reasoning
### Key concepts
- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius

#### Possible learning intentions
- Investigate properties of 2D shapes
- Investigate angles in polygons
- Understand and use the vocabulary of circles

### Possible success criteria
- Know the definitions of special triangles
- Know the definitions of special quadrilaterals
- Classify 2D shapes using given categories; e.g. number of sides, symmetry
- Know the angle sum of a triangle
- Know the angle sum of a quadrilateral
- Know how to find the angle sum of any polygon
- Use the angle sum of a triangle to find missing angles
- Find the missing angle in an isosceles triangle when only one angle is known
- Use the angle sum of a quadrilateral to find missing angles
- Know how to find the size of one angle in any regular polygon

### Prerequisites
- Know the properties of rectangles
- Know the difference between a regular and an irregular polygon
- Add and subtract numbers up to three digits

### Mathematical language
- Quadrilateral, Square, Rectangle, Parallelogram, (isosceles) Trapezium, Kite, Rhombus, Delta, Arrowhead
- Triangle, Scalene, Right-angled, Isosceles, Equilateral Polygon, Regular, Irregular
- Pentagon, Hexagon, Octagon, Decagon, Dodecagon
- Circle, Radius, Diameter, Circumference, Centre
- Parallel
- Diagonal
- Angle

#### Notation
- Dash notation to represent equal lengths in shapes and geometric diagrams
- Right angle notation

### Pedagogical notes
- Ensure that pupils do not use the word ‘diamond’ to describe a kite, or a square that is 45° to the horizontal. ‘Diamond’ is not the mathematical name of any shape.
- A square is a special case of a rectangle. An oblong is a rectangle that is not a square.
- A rhombus is a special case of a parallelogram.
- All polygons up to 20 sides have names, although many have alternatives based on either Latin or Greek.
- Splitting any polygon into triangles (by drawing all diagonals from one vertex) will allow pupils to find the angle sum of the polygon.
- NCETM: Glossary

#### Common approaches
- All teachers refer to a ‘delta’ instead of an ‘arrowhead’
- Every classroom has a set of triangle posters and quadrilateral posters on the wall

### Reasoning opportunities and probing questions
- Convince me that a rhombus is a parallelogram
- Jenny writes that ‘Diameter = 2 × Radius’. Kenny writes that ‘Radius = 2 × Diameter’. Who is correct?
- What is the same and what is different: a square and a rectangle?

### Suggested activities
- KM: Shape work: Many of the activities are suitable for this unit.
- KM: Dotty activities
- KM: Investigating polygons: Tasks one and two.
- KM: Special polygons
- NRICH: Where Are They?
- NRICH: Round a Hexagon
- NRICH: Quadrilaterals
- KM: 6 point circles, 8 point circles and 12 point circles can be used to support and extend the above idea
- Learning review: www.diagnosticquestions.com

### Possible misconceptions
- Some pupils may think that a ‘regular’ polygon is a ‘normal’ polygon
- Some pupils may think that all polygons have to be regular
- Some pupils may think that a square is only square if ‘horizontal’, and even that a ‘non-horizontal’ square is called a diamond
- The equal angles of an isosceles triangle are not always the ‘base angles’ as some pupils may think
Algebraic proficiency: using formulae

### 4 hours

#### Key concepts
- use simple formulae
- convert between miles and kilometres

#### The Big Picture:
- Algebra progression map

#### Possible learning intentions
- Use simple formulae written in words
- Create simple formulae written in words
- Work with formulae written algebraically

**Bring on the Maths**: Moving on up!

Algebra: #1

#### Possible success criteria
- Recognise a simple formula written in words
- Interpret the information given in a written formula
- Substitute numbers into a one-step formula written in words
- Substitute numbers into a two-step formula written in words
- Interpret the information that results from substituting into a formula
- Create a one-step formula from given information
- Create a two-step formula from given information
- Use symbols to represent variables in a formula

#### Prerequisites
- Know the order of operations
- Know the fact that area of rectangle = length \times width

#### Mathematical language
- Formula, Formulae
- Expression
- Variable
- Substitute
- Symbol
- Mile
- Kilometre
- Metric
- Imperial

**Notation**
- When written algebraically a formula should not include any units.

#### Pedagogical notes
- Pupils have already used the written formula 'area of rectangle = length \times width'. This can be used here to introduce the use of letters to represent variables; 'A = l \times w'. Later in the year pupils will meet other formulae for area and volume and this unit should be used to develop conceptual understanding in readiness for this. Other common examples that could be used include the rough conversion between miles and kilometres, 'kilometres = miles \times 1.6'.
- NCETM: Algebra
- NCETM: Glossary

#### Common approaches

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<td>Look at this formula. Write down a fact that it tells you. And another. And another ... Jenny and Kenny are using the formula ‘Cost in pounds = 40 + 20 \times number of hours’ to work out the cost for three hours. Jenny writes down £180. Kenny writes down £100. Who do you agree with? Why? Always / Sometimes / Never: The formula T = 4n + 6 results in an odd number.</td>
<td>KM: Fascinating food, NCETM: Year 6 Algebra. Activities A and D. Learning review <a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a></td>
<td>Some pupils may apply the order of operations incorrectly when working with two step formulae. Units must be consistent when using formulae. For example, a mobile phone plan might charge £15 per month plus 5p for every text. The formula ‘Monthly cost = 15 + 5 \times number of texts’ is wrong because amounts in both pounds and pence are involved. Monthly cost (in pence) = 1500 + 5 \times number of texts is one correct way of writing the formula. It is not advisable to abbreviate the formula ‘kilometres = miles \times 1.6’ using letters. ‘m’ is the normal abbreviation for metres and ‘k’ can represent £1000. If ‘km’ is used it could even be interpreted as ‘k \times m’.</td>
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### Key concepts
- Use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- Compare and order fractions, including fractions > 1
- Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, \( \frac{3}{8} \)]
- Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts

#### The Big Picture: Fractions, decimals and percentages progression map

### Possible learning intentions
- Explore the equivalence between fractions
- Use the equivalence between fractions
- Explore the equivalence between fractions, decimals and percentages

#### Bring on the Maths: Moving up!
- Fractions, decimals & percentages: #1, #2

### Possible success criteria
- Understand that two fractions can be equivalent
- Identify a common factor of two numbers
- Simplify a fraction
- Write a fraction in its lowest terms
- Confirm that a fraction is written in its lowest terms
- Compare two fractions by considering diagrams
- Compare two fractions by considering equivalent fractions
- Compare two top-heavy fractions
- Understand that a fraction is also a way of representing a division
- Know standard fraction / decimal equivalences (e.g. \( \frac{1}{2} = 0.5 \), \( \frac{1}{4} = 0.25 \), \( \frac{1}{10} = 0.1 \))
- Work out the decimal equivalents of fifths, eighths and tenths
- Know standard fraction / decimal / percentage equivalences (e.g. 10%, 25%, 50%, 75%)
- Work out the percentage equivalents of fifths, eighths and tenths
- Use the equivalence between fractions, decimals and percentages when solving problems

### Prerequisites
- Understand the concept of a fraction as a proportion
- Understand the concept of equivalent fractions
- Understand the concept of fractions, decimals and percentages being equivalent
- Know that a percentage means ‘out of 100’

#### Mathematical language
- Fraction
- Improper fraction, Proper fraction, Vulgar fraction, Top-heavy fraction
- Percentage
- Decimal
- Proportion
- Simplify
- Equivalent
- Lowest terms

#### Pedagogical notes
- Use language carefully to avoid later confusion: when simplifying fractions, the language ‘divide by 4’ should not be used in place of ‘divide the top and bottom by 4’. A fraction can be divided by 4, but that is not the same as cancelling a common factor of the numerator and denominator by dividing them by 4.
- NRICH: Teaching fractions with understanding
- NCETM: Teaching fractions
- NCETM: Glossary

#### Common approaches
- All pupils are made aware that ‘per cent’ is derived from Latin and means ‘out of one hundred’
- Teachers use the horizontal fraction bar notation at all times

### Reasoning opportunities and probing questions
- Show me another fraction that is equivalent to this one. And another. And another ...
- Convince me that \( \frac{3}{8} = 0.375 \)
- If you know that \( \frac{1}{10} = 0.1 \), what else can you work out?
- Jenny is simplifying fractions. She has the fraction \( \frac{16}{64} \). Jenny says, ‘if I cancel out the sixes then \( \frac{16}{64} = \frac{1}{4} \).’ Do you agree with Jenny? Why?

#### Suggested activities
- KM: FDP conversion
- KM: Carpets
- KM: Fraction and decimal tables
- NRICH: Matching fractions
- NRICH: Fractions made faster

#### Learning review
- www.diagnosticquestions.com

### Possible misconceptions
- A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape.
- Pupils may not make the connection that a percentage is a different way of describing a proportion
- Some pupils may think that simplifying a fraction just requires searching for, and removing, a factor of 2 (repeatedly)
## Proportional reasoning

### Key concepts
- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

### The Big Picture
- Ratio and Proportion progression map

### Possible learning intentions
- Solve problems involving scaling
- Explore enlargement
- Solve problems involving sharing and grouping

### Possible success criteria
- Identify when a comparison problem can be solved using multiplication
- Identify when a comparison problem can be solved using division
- Identify when a comparison problem requires both division and multiplication
- Find the value of a single item in a comparison problem
- Use the value of a single item to solve a comparison problem
- Understand the meaning of enlargement
- Understand the meaning of scale factor
- Recognise when one shape is an enlargement of another
- Use a scale factor to complete an enlargement
- Find the scale factor for a given enlargement
- Use knowledge of fractions to solve a sharing (or grouping) problem
- Use knowledge of multiples to solve a sharing (or grouping) problem

### Prerequisites
- Recall multiplication facts for multiplication tables up to 12 × 12
- Recall division facts for multiplication tables up to 12 × 12
- Find fractions of an amount
- Find multiples of a given number

### Mathematical language
- Proportion
- Quantity
- Integer
- Similar (shapes)
- Enlargement
- Scale factor
- Group
- Share
- Multiples

### Pedagogical notes
- Any work on enlargement should only include enlargements using a scale factor. The concept of a centre of enlargement is a future development.

### Reasoning opportunities and probing questions
- (Given a recipe for 4 people) show me an amount of food that is needed for 8 people, 6 people, 9 people. Show me an amount of food that is needed for a number of people of your choice. And another. And another ...
- Convince me that the second shape is an enlargement of the first shape
- Kenny has no sweets. Jenny gives 1/3 of her sweets to Kenny. Kenny now has 18 sweets. Kenny thinks that Jenny had 54 sweets to start with. Kenny is wrong. Explain why.

### Possible misconceptions
- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts
- When finding a fraction of an amount some pupils may try to use a rule formed without the necessary understanding. As a result they will muddle the operations, dividing by the numerator and multiplying by the denominator.
- When constructing an enlargement some pupils may only apply the scale factor in one dimension; for example, ‘enlarging’ a 2 by 4 rectangle by a scale factor of 2 and drawing a 2 by 8 rectangle.

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Return to overview

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### Bring on the Maths: Moving on up!
- Ratio and proportion: #1
- Y7 Bring on the Maths
- Problem Solving: #1, #2, #3

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### Possible misconceptions
- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts
- When finding a fraction of an amount some pupils may try to use a rule formed without the necessary understanding. As a result they will muddle the operations, dividing by the numerator and multiplying by the denominator.
- When constructing an enlargement some pupils may only apply the scale factor in one dimension; for example, ‘enlarging’ a 2 by 4 rectangle by a scale factor of 2 and drawing a 2 by 8 rectangle.

---

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## Pattern sniffing

**Key concepts**
- generate and describe linear number sequences

### Possible learning intentions
- **Bring on the Maths**: Moving on up!
  - Number and Place Value: #4
  - Number and Place Value: #5

### Possible success criteria
- use the vocabulary of sequences
- recognise a linear sequence
- describe a number sequence
- find the next term in a linear sequence
- find a missing term in a linear sequence
- generate a linear sequence from its description

### Prerequisites
- count forwards and backwards in tens (hundreds, thousands) from any positive number up to 10 000 (100 000, 1 000 000)
- count forwards and backwards through zero

### Mathematical language
- Pattern
- Sequence
- Linear
- Term
- Ascending
- Descending

### Pedagogical notes
- Pupils have counted forwards and backwards in previous years and units, but this is the first time that the concept of sequences appears specifically.
- The language ‘term-to-term rule’ should not be introduced until Stage 7.

### Common approaches
- Teachers and pupils refer to numbers less than zero as ‘negative’ numbers and not ‘minus’ numbers

### Reasoning opportunities and probing questions
- show me a (ascending/descending) linear sequence. and another. and another.
  - kenny thinks that 2, 4, 8, 16, … is a linear example. do you agree? explain your answer.
  - create a linear sequence with a 3rd term of ‘8’.
  - show me a linear sequence where the rule to get from one term to the next is ‘add 3’. and another. and another.

### Suggested activities
- KM: Maths to Infinity: Sequences
- NRICH: Times Tables Shifts
- NRICH: Domino Sets
- NCETM: Activity B: Sticky Triangles
- NCETM: Activity D: Generating Sequences
- Learning review: www.diagnosticquestions.com

### Possible misconceptions
- Some pupils may think linear sequences are only ascending.
- Some pupils may think that any sequence that can be described by a rule to get from one term to the next is a linear sequence, e.g. 2, 4, 8, 16, …
- Some pupils may not appreciate that both a starting number and a rule to find the next term are required in order to describe a sequence in full.
### Measuring space

**Key concepts**
- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places

**The Big Picture:** Measurement and mensuration progression map

**Possible learning intentions**
- Solve problems involving measurement

**Possible success criteria**
- Convert between non-adjacent metric units; e.g. kilometres and centimetres
- Use decimal notation up to three decimal places when converting metric units
- Convert between Imperial units; e.g. feet and inches, pounds and ounces, pints and gallons
- Solve problems involving converting between measures
- State conclusions using the correct notation and units

**Prerequisites**
- Convert between adjacent metric units of length, mass and capacity
- Know rough equivalents between inches and cm, feet and cm, kg and lb, pint and ml
- Use decimal notation to two decimal places when converting between metric units

**Mathematical language**
- Length, distance
- Mass, weight
- Volume
- Capacity
- Metre, centimetre, millimetre
- Tonne, kilogram, gram, milligram
- Litre, millilitre
- Hour, minute, second
- Inch, foot, yard
- Pound, ounce
- Pint, gallon

**Notation**
- Abbreviations of units in the metric system: m, cm, mm, kg, g, l, ml
- Abbreviations of units in the Imperial system: lb, oz

**Pedagogical notes**
- Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement. The prefix ‘centi-’ means one hundredth, and the prefix ‘milli-’ means one thousandth. These words are of Latin origin. The prefix ‘kilo-’ means one thousand. This is Greek in origin. Conversion of volumes will be covered in the ‘calculating space’ unit.
- NCTM: Glossary

**Common approaches**
- Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)

**Reasoning opportunities and probing questions**
- Show me a metric (imperial) unit of measure. And another. And another.
- Kenny thinks that 2.5km = 25 000 cm. Do you agree with Kenny? Explain your answer.
- Convince me that 4.25kg does not equal 425g.

**Suggested activities**
- KM: Weighing up the options
- NRICH: Place Your Orders
- NRICH: Thousands and Millions
- NCTM: Activity E: A little bit of history - Marco Polo

**Possible misconceptions**
- Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems
- Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o’clock
- Some pupils may apply incorrect beliefs about place value, such as $2.3 \times 10 = 2.30$
- Many conversions within the metric system rely on multiplying and dividing by 1000. The use of centimetres as an ‘extra unit’ within the system breaks this pattern. Consequently there is a frequent need to multiply and divide by 10 or 100, and this can cause confusion about the connections that need to be applied.

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**Possible misconceptions**
### Key concepts
- recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles

### The Big Picture
- Position and direction progression map

### Possible learning intentions
- Develop knowledge of angles
- Apply angle facts to deduce unknown angles

### Possible success criteria
- Identify angles that meet at a point
- Identify angles that meet at a point on a line
- Identify vertically opposite angles
- Know that vertically opposite angles are equal
- Use known facts to find missing angles
- Explain reasoning

### Prerequisites
- Know that angles are measured in degrees
- Know that angles in a full turn total 360°, and angle in half a turn must total 180°
- Estimate the size of angles

### Mathematical language
- Angle
- Degrees
- Right angle
- Acute angle
- Obtuse angle
- Reflex angle
- Protractor
- Vertically opposite

#### Notation
- Right angle notation
- Arc notation for all other angles
- The degree symbol (°)

### Pedagogical notes
- The exact reason for there being 360 degrees in a full turn is unknown. There are various theories including it being an approximation of the 365 days in a year and resultant apparent movement of the sun, and the fact that it has so many factors.
- The SI unit for measuring angles is the radian (2π radians in a full turn).
- Napoleon experimented with the decimal degree, or grad (400 grads in a full turn).

### Reasoning opportunities and probing questions

#### Suggested activities
- Show a pair of possible values for a and b. And another. And another.
- Convince me that the sum of angles on a straight line is 180°. Show a possible set of values for a, b, c and d. And another. And another.
- Convince me that the sum of angles around a point is 360°.
- Convince me that (vertically) opposite angles are equal.
- Kenny thinks that the sum of opposite angles is 180°. Do you agree? Explain your answer.

### Possible misconceptions
- Some pupils may think that these angles are not equal as they are not 'vertical'.
- Some pupils may think that angles that are 'roughly' opposite are always equal, e.g. a = c
## Key concepts
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, \( \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \)]
- divide proper fractions by whole numbers [for example, \( \frac{1}{5} \div 2 = \frac{1}{10} \)]
- multiply one-digit numbers with up to two decimal places by whole numbers
- solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison

### Possible learning intentions
- Calculate with fractions
- Calculate with decimals
- Calculate with percentages

### Possible success criteria
- Add (subtract) fractions with different denominators
- Add (subtract) a mixed number and a fraction, including with different denominators
- Add (subtract) mixed numbers, including with different denominators
- Multiply a proper fraction by a proper fraction
- Divide a proper fraction by a whole number
- Simplify the answer to a calculation when appropriate
- Multiply U.t by U
- Multiply U.t by U
- Find 10% of a quantity
- Use non-calculator methods to find a percentage of an amount
- Use decimal or fraction equivalents to find a percentage of an amount where appropriate
- Solve problems involving the use of percentages to make comparisons

### Prerequisites
- Convert between mixed numbers and improper fractions
- Find equivalent fractions
- Add and subtract fractions when one denominator is a multiple of the other
- Multiply a proper fraction by a whole number
- Use the formal written method of short multiplication
- Know the effect of multiplying and dividing by 10 and 100
- Know percentage equivalents of \( \frac{1}{5}, \frac{1}{4}, \frac{1}{2}, \frac{3}{5}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6} \)

### Mathematical language
- Mixed number
- Equivalent fraction
- Simplify, cancel
- Lowest terms
- Proper fraction, improper fraction, top-heavy fraction, vulgar fraction
- Numerator, denominator
- Percent, percentage
- Notation
  - Mixed number notation
  - Horizontal / diagonal bar for fractions

### Pedagogical notes
- Use of a fraction wall to visualise multiplying fractions and dividing fractions by a whole number. For example, pupils need to read calculations such as \( \frac{1}{4} \times \frac{1}{2} \) as \( \frac{1}{4} \) multiplied by \( \frac{1}{2} \) and therefore, \( \frac{1}{4} \) of \( \frac{1}{2} = \frac{1}{8} \)
- \( \frac{4}{10} + \frac{2}{10} \) as \( \frac{4}{10} \) divided by 2 and therefore \( \frac{2}{10} \)
- NCETM: The Bar Model
- NCETM: Teaching fractions
- NCETM: Fractions videos
- NCETM: Glossary

### Common approaches
When multiplying a decimal by a whole number pupils are taught to use the corresponding whole number calculation as a general strategy. When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy. Pupils are encouraged to find and use 10% of an amount.

### Reasoning opportunities and probing questions
- Show me an ‘easy’ (‘difficult’) pair of fractions to add (subtract). And another. And another.
  - Kenny thinks that \( \frac{7}{10} - \frac{2}{5} = \frac{5}{10} \). Do you agree with Kenny?
  - Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain.
  - Benny thinks that \( \frac{4}{10} + 2 = \frac{4}{10} \). Do you agree with Benny? Explain.
  - Lenny says ‘20% of £60 is £3 because 60 ÷ 20 = 3’. Do you agree?

### Learning review
- www.diagnosticquestions.com

### Suggested activities
- NRICH: Fractions Jigsaw
- NRICH: Peaches Today, Peaches Tomorrow
- NRICH: Andy’s Marbles
- NRICH: Would you Rather?

### Possible misconceptions
- Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional part of mixed numbers when adding/subtracting mixed numbers, e.g. \( 3\frac{1}{2} - 2\frac{1}{2} = 1\frac{1}{2} \)
- Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators.
- Some pupils may think that as you divide by 10 to find 10%, you divided by 15 to find 15%, divide by 20 to find 20%, etc.
## Solving equations and inequalities

**4 hours**

### Key concepts
- enumerate possibilities of combinations of two variables
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns

<table>
<thead>
<tr>
<th>Possible learning intentions</th>
<th>Possible success criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve missing number problems</td>
<td>Solve missing number problems expressed in words</td>
</tr>
<tr>
<td>Understand and use algebra</td>
<td>Find a solution to a missing number problem with two unknowns</td>
</tr>
<tr>
<td></td>
<td>Find all combinations of two variables that solve a missing number problem with two unknowns</td>
</tr>
<tr>
<td></td>
<td>Know the basic rules of algebraic notation</td>
</tr>
<tr>
<td></td>
<td>Express missing number problems algebraically</td>
</tr>
<tr>
<td></td>
<td>Solve missing number problems expressed algebraically</td>
</tr>
</tbody>
</table>

### Possible misconceptions
- Some pupils may think that variables have a set value, such as \(a = 1, b = 2, c = 3, d = 4\), etc. (especially if they have done lots of poorly designed treasure hunts/codes) – this will lead to problems such as thinking \(b^2\) is the same as \(2b\) because when \(b = 2\), \(b^2 = 4\) and \(2b = 4\). Do you agree with Jenny? Explain your answer.
- Using the idea of ‘apples’ and ‘bananas’ to explain \(a + b = 14\) can lead to misconceptions about the use of letters as variables.
- Some students may think that the variables have to be positive integers (whole numbers).

### Bring on the Maths: Moving on up!

**Algebra: #2**

### Prerequisites
- Use symbols to represent variables in a formula

### Mathematical language
- Algebra, algebraic, algebraically
- Symbol
- Expression
- Variable
- Substitute
- Equation
- Unknown
- Enumerate

### Notation
- The lower case and upper case of a letter should not be used interchangeably when worked with algebra.
- Juxtaposition is used in place of ‘\(\times\)’. \(2a\) is used rather than \(a2\).
- Division is written as a fraction.

### Pedagogical notes
- The word ‘algebra’ comes from the title of a book by the Persian mathematician, al-Khwārizmī, who lived in modern-day Baghdad about 1200 years ago. Al-kitāb al-mukhtaṣar fī ḥisāb al-ğabr wa'l-muqābala was a book that promoted the idea of solving equations by a method of balancing.
- Avoid fruit salad algebra (see possible misconceptions).
- NCETM: The Bar Model
- NCETM: Algebra
- NCETM: Glossary

### Reasoning opportunities and probing questions
- \(a + b = 15\). Show me a pair of values for \(a\) and \(b\). And another. And another.
- \(p + q = 7\). Show me a pair of values for \(p\) and \(q\) that no one else will think of. And another. And another.
- Kenny thinks that \(b^2\) is the same as \(2b\) because when \(b = 2\), \(b^2 = 4\) and \(2b = 4\). Do you agree with Kenny? Explain your answer.
- Jenny thinks that \(7 + 2a = 9a\). Do you agree with Jenny? Explain your answer.

### NCETM: Algebra Reasoning

### Reasoning
- Suggested activities
- NRICH: Plenty of Pens
- NRICH: Your Number Is...
- NRICH: Number Pyramids
- NCETM: Activity A: Racetrack and Design a board game
- NCETM: Activity E: Matchbox Algebra

### Learning review
- www.diagnosticquestions.com
### Calculating space

#### Key concepts
- recognise that shapes with the same areas can have different perimeters and vice versa
- calculate the area of parallelograms and triangles
- calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm³) and cubic metres (m³), and extending to other units (for example, mm³ and km³)
- recognise when it is possible to use formulae for area and volume of shape
- solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate

<table>
<thead>
<tr>
<th>Possible learning intentions</th>
<th>Possible success criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explore area</td>
<td>Recognise that shapes with the same areas can have different perimeters and vice versa</td>
</tr>
<tr>
<td>Investigate volume</td>
<td>Know that the area of a parallelogram is given by the formula area = base \times height</td>
</tr>
<tr>
<td>Solve problems involving area and volume</td>
<td>Know that the area of a triangle is given by the formula area = \frac{1}{2} \times base \times height</td>
</tr>
<tr>
<td><strong>Bring on the Maths</strong>: Moving on up!</td>
<td>Know that the volume of a cuboid is given by the formula volume = length \times width \times height</td>
</tr>
</tbody>
</table>

| Measures: #6 | **Bring on the Maths**: Moving on up! Measures: #4, #5 |

<table>
<thead>
<tr>
<th>Prerequisites</th>
<th>Mathematical language</th>
<th>Pedagogical notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Know the meaning of perimeter (area, volume, capacity)</td>
<td>Perimeter, area, volume, capacity</td>
<td>In this unit, 'volumes of shapes' refers only to cubes and cuboids. Ensure that pupils make connections with the area of a rectangle work in Stage 5, in particular the importance of the perpendicular height. Note that there are several different ways of stating the area of a triangle and this can cause confusion.</td>
</tr>
<tr>
<td>Know that the area of a rectangle is given by the formula area = length \times width</td>
<td>Square, rectangle, parallelogram, triangle</td>
<td>NCETM: Glossary</td>
</tr>
<tr>
<td>Know that area can be measured using square centimetres or square metres, and the abbreviations cm² and m²</td>
<td>Composite rectilineal</td>
<td>Common approaches</td>
</tr>
<tr>
<td>Know that volume is measured in cubes</td>
<td>Polygon</td>
<td>Pupils derive practically the formulae for area of parallelogram and triangle by dissecting rectangles</td>
</tr>
<tr>
<td>Know that area can be measured using square centimetres or square metres, and the abbreviations cm² and m²</td>
<td>Cube, cuboid</td>
<td>Pupils derive the formula for the area of a parallelogram first. They then use this to help derive the formula for the area of an obtuse-angled triangle.</td>
</tr>
<tr>
<td>Know that area can be measured in cubes</td>
<td>Millimetre, Centimetre, Metre, Kilometre</td>
<td>Every classroom has a set of area posters on the wall</td>
</tr>
<tr>
<td>Know that area can be measured using square centimetres or square metres, and the abbreviations cm² and m²</td>
<td>Square millimetre, square centimetre, square metre, square kilometre</td>
<td>Pupils use the area of a triangle as given by the formula area = \frac{bh}{2}</td>
</tr>
<tr>
<td>Know that volume is measured in cubes</td>
<td>Cubic centimetre, centimetre cube</td>
<td></td>
</tr>
<tr>
<td>Know that volume is measured in cubes</td>
<td>Formula, formulae</td>
<td></td>
</tr>
<tr>
<td>Know that volume is measured in cubes</td>
<td>Convert</td>
<td></td>
</tr>
<tr>
<td>Know that volume is measured in cubes</td>
<td>Length, breadth, depth, height, width</td>
<td></td>
</tr>
</tbody>
</table>

| Notation | Abbreviations of units in the metric system: km, m, cm, mm, mm², cm², m², km², mm³, cm³, km³ |

<table>
<thead>
<tr>
<th>Reasoning opportunities and probing questions</th>
<th>Suggested activities</th>
<th>Possible misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘Show me’ an example of when you would measure volume using km³</td>
<td>KM: Fibonacci’s disappearing squares</td>
<td></td>
</tr>
<tr>
<td>Convince me that the area of a parallelogram is found using base \times height</td>
<td>KM: Dissections deductions</td>
<td></td>
</tr>
<tr>
<td>(Given a triangle with base labelled 8 cm, height 5 cm, slope height 6 cm) Jenny thinks that the area is 40 cm², Lenny thinks it is 20 cm², Penny thinks it is 240 cm² and Benny thinks it is 24 cm². Who do you agree with? Explain why.</td>
<td>KM: Stick on the Maths SSM9: Area and volume</td>
<td></td>
</tr>
<tr>
<td>KM: Maths to Infinity Area and Volume</td>
<td>KM: Maths to Infinity Area and Volume</td>
<td></td>
</tr>
<tr>
<td>KCETM: Activity C: Through the window</td>
<td>NCETM: Activity C: Through the window</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Learning review</th>
<th>Suggested activities</th>
<th>Possible misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td><a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a></td>
<td>KM: Fibonacci’s disappearing squares</td>
<td>Some pupils may use the sloping height when finding the areas of parallelograms and triangles</td>
</tr>
<tr>
<td>KM: Dissections deductions</td>
<td>KM: Stick on the Maths SSM9: Area and volume</td>
<td>Some pupils may think that the area of a triangle is found using area = base \times height</td>
</tr>
<tr>
<td>KM: Maths to Infinity Area and Volume</td>
<td>KM: Maths to Infinity Area and Volume</td>
<td>Some pupils may think that you multiply all the numbers to find the area of a shape</td>
</tr>
<tr>
<td>NCETM: Activity C: Through the window</td>
<td>NCETM: Activity C: Through the window</td>
<td></td>
</tr>
</tbody>
</table>

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**NCETM: Geometry - Properties of Shapes Reasoning**

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www.kangaroomaths.com
### Checking, approximating and estimating

**4 hours**

#### Key concepts
- solve problems which require answers to be rounded to specified degrees of accuracy
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- round any whole number to a required degree of accuracy

#### The Big Picture:
- Number and Place Value progression map

#### Possible learning intentions
- Explore ways of approximating numbers
- Explore ways of checking answers

#### Possible success criteria
- Approximate any number by rounding to the nearest 1 000 000
- Approximate any number by rounding to a specified degree of accuracy; e.g. nearest 20, 50
- Understand estimating as the process of finding a rough value of an answer or calculation
- Use estimation to predict the order of magnitude of the solution to a (decimal) calculation
- Check the order of magnitude of the solution to a (decimal) calculation
- Estimate multiplication calculations that involve multiplying up to four-digit numbers by a two-digit number
- Estimate division calculations that involve dividing up to a four-digit number by a two-digit number
- Estimate multiplication calculations that involve multiplying numbers with up to two decimal places by whole numbers

#### Prerequisites
- Approximate any number by rounding to the nearest 10, 100 or 1000, 10 000 or 100 000
- Approximate any number with one or two decimal places by rounding to the nearest whole number
- Approximate any number with two decimal places by rounding to the one decimal place
- Estimate addition (subtraction) calculations with up to four digits

#### Mathematical language
- Approximate (noun and verb)
- Round
- Decimal place
- Check
- Solution
- Answer
- Estimate (noun and verb)
- Order of magnitude
- Accurate
- Accuracy

#### Notation
- The approximately equal symbol (\(\approx\))

#### Pedagogical notes
- This unit is an opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.
- Pupils should use numbers up to 10 000 000 in this unit.
- Pupils should be able to round to other specified degrees of accuracy, but not to a specified number of significant figures, which is introduced in Stage 7.
- Also see big pictures: Calculation progression map and Fractions, decimals and percentages progression map
- NCETM: Glossary

#### Common approaches
- All pupils are taught to visualise rounding through the use a number line

#### Reasoning opportunities and probing questions
- Convince me that 67 rounds to 60 to the nearest 20
- Convince me that 1 579 234 rounds to 2 million to the nearest million
- Jenny writes 1359 ÷ 18 \(\approx\) 7.55. Comment on Jenny’s approximation.
- Lenny writes 2.74 × 13 \(\approx\) 26. Do you agree with Lenny? Explain your answer.
- Some pupils may truncate instead of round
- When checking the order of magnitude of a division calculation some pupils may apply incorrect reasoning about the effect of increasing the divisor by a factor of 10, thinking that it also makes the solution greater by a factor of 10; e.g. 1400 ÷ 20: 1400 ÷ 2 = 700 so 1400 ÷ 20 = 7000.
- Some pupils may round down at the half way point, rather than round up.

#### Suggested activities
- KM: Stick on the Maths CALC6: Checking results
- KM: Maths to Infinity Rounding
- NRICH: Four Go
- NCETM: Activity A(i)
- NCETM: Activity G

#### Learning review
- www.diagnosticquestions.com
### Mathematical movement

#### Key concepts
- describe positions on the full coordinate grid (all four quadrants)
- draw and translate simple shapes on the coordinate plane, and reflect them in the axes

#### The Big Picture: Position and direction progression map

<table>
<thead>
<tr>
<th>Possible learning intentions</th>
<th>Possible success criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand and use Cartesian coordinates</td>
<td>Use coordinates to describe the position of a point in all four quadrants</td>
</tr>
<tr>
<td>Use transformations to move shapes</td>
<td>Use coordinates to write the position of a point in all four quadrants</td>
</tr>
</tbody>
</table>

#### Prerequisites
- Use coordinates in the first quadrant
- Identify a translation
- Carry out a translation in the first quadrant
- Identify a reflection
- Carry out a reflection in the first quadrant using mirror lines parallel to the axes
- Know the meaning of ‘congruent’, ‘congruence’, ‘object’, ‘image’

#### Mathematical language
- 2-D
- Grid
- Axis, axes, x-axis, y-axis
- Origin
- Quadrant
- (Cartesian) coordinates
- Point
- Translation
- Reflection
- Transformation
- Object, Image
- Congruent, congruence

#### Notation
- Cartesian coordinates should be separated by a comma and enclosed in brackets \((x, y)\)

#### Pedagogical notes
- The main focus of this unit is to develop understanding of coordinates in all four quadrants.
- Note that pupils are not yet expected to use an algebraic description of a mirror line (such as \(x = 3\)).
- The French mathematician Rene Descartes introduced Cartesian coordinates in the 17th century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling.
- Other coordinate systems include grid references, polar coordinates and spherical coordinates.
- There are other types of mathematical movement that pupils will learn about in future stages. The group name for these movements is ‘transformations’.

#### Common approaches
- Teachers do not use the phrase ‘along the corridor and up the stairs’ as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants.
- A more helpful way to remember the order of coordinates is ‘x is a cross, wise up!’
- Teachers use the language ‘negative number’, and not ‘minus number’, to avoid future confusion with calculations.

#### Reasoning opportunities and probing questions
- **(Given a grid with the point \((-3, 4)\) indicated)** Benny describes this point as \((-3, 4)\). Jenny describes the point as \((4, -3)\). Who do you agree with? Why?
- **Two vertices of a rectangle are \((-1, 2)\) and \((4, -2)\). What could the other two vertices be?** How many solutions can you find?
- **Convince me that \((-2, 3)\) is in the second quadrant**

#### Suggested activities
- **KM:** Stick on the Maths ALG2: Coordinates in four quadrants
- **NRICH:** Cops and Robbers
- **NRICH:** Eight Hidden Squares
- **NRICH:** Coordinate Tan
- **NRICH:** Transformation Tease
- **NCETM:** Activity B - Battleships

#### Learning review
- [www.diagnosticquestions.com](http://www.diagnosticquestions.com)

#### Possible misconceptions
- When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
- When reflecting a triangle some students may draw a translation.
- When carrying out a reflection some pupils may think that the object and image should be an equal distance from the edge of the grid, rather than an equal distance from the mirror line.
- Some pupils will confuse the order of x-coordinates and y-coordinates.
- When constructing axes, some pupils may not realise the importance of equal divisions on the axes.
**Key concepts**

- Interpret and construct pie charts and line graphs and use these to solve problems

### Possible learning intentions

- Construct and interpret pie charts
- Solve problems involving graphs and charts

**Bring on the Maths**: Moving on up!

Statistics: #2, #3

### Possible success criteria

- Understand that pie charts are used to show proportions
- Make statements about proportions shown in a pie charts
- Make statements to compare proportions in pie charts
- Use additional information to make statements about frequencies in pie charts
- Use a table of frequencies to work out the angle for a slice in a pie chart
- Construct a pie chart by measuring angles
- Identify the scale used on the axes of a graph
- Read values from a line graph involving scaling
- Use scaling when constructing line graphs
- Answer two-step questions about data in line graphs (e.g. ‘How much more?’)

### Prerequisites

- Measure and construct angles using a protractor
- Interpret and construct a simple line graph

### Mathematical language

- Data
- Scale
- Axis, axes
- Graph
- Frequency
- Time graph, Time series
- Line graph
- Pie chart
- Sector
- Angle
- Protractor
- Degrees
- Maximum, minimum

### Pedagogical notes

- In Stage 6, when constructing pie charts the total of the frequencies is always a factor of 360. More complex cases are included in later stages.

William Playfair, a Scottish engineer and economist, introduced the line graph in 1786. He also introduced the pie chart in 1801.

NCETM: Glossary

**Common approaches**

Pie charts are constructed by calculating the angle for each section by dividing 360 by the total frequency, and not using percentages. The angle for the first section is measured from a vertical radius. Subsequent sections are measured using the boundary line of the previous section.

### Reasoning opportunities and probing questions

- Show me a pie chart representing the following information: Blue (25%), Red (over 50%), Yellow (the rest). And another. And another.

**Suggested activities**

- Always / Sometimes / Never: Pie charts are constructed in a clockwise direction
- Always / Sometimes / Never: The larger the size of the pie chart, the greater the total frequency
- Kenny says ‘If two pie charts have the same section then the amount of data the section represents is the same in each pie chart.’ Do you agree with Kenny? Explain your answer.

NCETM: Statistics Reasoning

**Possible misconceptions**

- Some pupils may think that a line graph is appropriate for discrete data
- Some pupils may think that each square on the grid used represents one unit
- Some pupils may confuse the fact that the sections of the pie chart total 100% and 360°

NCETM: Graphs and diagrams

NRICH: Match the Matches

NRICH: Graphing Number Patterns

NCETM: A little bit of history (Britain since 1945)

Learning review

www.diagnosticquestions.com
### Measuring data

**Key concepts**
- calculate and interpret the mean as an average

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### Possible learning intentions
- Understand and use the mean

### Possible success criteria
- Understand the meaning of ‘average’ as a typicality (or location)
- Understand the mean as a measure of typicality (or location)
- Interpret the mean as a way of levelling the data
- Calculate the mean of a set of data
- Choose an appropriate approximation when required
- Use the mean to find a missing number in a set of data

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### Prerequisites
- Approximate a number by rounding to a given number of decimal places

### Mathematical language
- Average
- Mean
- Measure
- Data
- Statistic
- Statistics
- Approximate
- Round

### Pedagogical notes
- The word ‘average’ is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the ‘arithmetic mean’). Other types of average, including the mode and the median, are introduced in later stages.
- NCETM: [Glossary](https://www.ncetm.org.uk/)

### Common approaches
- Always use brackets when writing out the calculation for a mean, e.g. \((2 + 3 + 4 + 5) ÷ 4 = 14 ÷ 4 = 3.5\)

### Reasoning opportunities and probing questions
- Always / Sometimes / Never: The mean is a whole number.
- Kenny is working out the mean of 2, 3, 4 and 5. He calculates \(2 + 3 + 4 + 5 + 4 = 10.25\). Do you agree with Kenny? Explain your answer.
- The average number of children per family (Married Couples, 2012) is 1.8. Convince me that this statement makes sense.

### Suggested activities
- KM: [Maths to Infinity: Averages, Charts and Tables](https://www.kangaroomaths.com)
- NRICH: [Birdwatch](https://nrich.maths.org/)
- NRICH: [Probably ...](https://nrich.maths.org/)
- NRICH: [Same or Different?](https://nrich.maths.org/)
- NCETM: [A little bit of history (Britain since 1945)](https://www.ncetm.org.uk/)
- Learning review [www.diagnosticquestions.com](http://www.diagnosticquestions.com)

### Possible misconceptions
- If using a calculator some pupils may not use the ‘=’ symbol (or brackets) correctly; e.g. working out the mean of 2, 3, 4 and 5 as \(2 + 3 + 4 + 5 ÷ 4 = 10.25\).
- Some pupils may think the average is always the middle number.
- Some pupils may think that the mean must be a whole number.
- Some pupils may not realise that the mean must lie within the range of the data set.