## Mathematics overview: Stage 5

<table>
<thead>
<tr>
<th>Unit</th>
<th>Hours</th>
<th>Mastery indicators</th>
<th>Essential knowledge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers and the number system</td>
<td>8</td>
<td>• Identify multiples and factors of a number</td>
<td>• Know the place value headings up to millions</td>
</tr>
<tr>
<td>Counting and comparing</td>
<td>8</td>
<td>• Count forwards and backwards through zero</td>
<td>• Recall primes to 19</td>
</tr>
<tr>
<td>Calculating: addition and subtraction</td>
<td>8</td>
<td>• Round to one decimal place</td>
<td>• Know the first 12 square numbers</td>
</tr>
<tr>
<td>Calculating: multiplication and division</td>
<td>12</td>
<td>• Use columnar addition and subtraction with numbers of any size</td>
<td>• Know the Roman numerals I, V, X, L, C, D, M</td>
</tr>
<tr>
<td>Investigating properties of shapes</td>
<td>4</td>
<td>• Multiply a three- or four-digit number by a two-digit number using long multiplication</td>
<td>• Know percentage and decimal equivalents for 1/2, 1/4, 1/5, 2/5, 4/5</td>
</tr>
<tr>
<td>Visualising and constructing</td>
<td>4</td>
<td>• Divide numbers up to four-digits by a single-digit number using short division and interpret the remainder</td>
<td>• Know rough conversions between metric and Imperial units</td>
</tr>
<tr>
<td>Exploring time</td>
<td>4</td>
<td>• Add and subtract fractions with denominators that are multiples of the same number</td>
<td>• Know that angles are measured in degrees</td>
</tr>
<tr>
<td>Exploring fractions, decimals and percentages</td>
<td>12</td>
<td>• Write decimals as fractions</td>
<td>• Know angles in one whole turn total 360°</td>
</tr>
<tr>
<td>Pattern sniffing</td>
<td>4</td>
<td>• Understand that per cent relates to number of parts per hundred</td>
<td>• Know angles in half a turn total 180°</td>
</tr>
<tr>
<td>Measuring space</td>
<td>8</td>
<td>• Convert between adjacent metric units of measure for length, capacity and mass</td>
<td>• Know that area of a rectangle = length × width</td>
</tr>
<tr>
<td>Investigating angles</td>
<td>8</td>
<td>• Measure and draw angles</td>
<td></td>
</tr>
<tr>
<td>Calculating fractions, decimals and percentages</td>
<td>12</td>
<td>• Distinguish between regular and irregular polygons</td>
<td></td>
</tr>
<tr>
<td>Calculating space</td>
<td>8</td>
<td>• Stage 5 BAM Progress Tracker Sheet</td>
<td></td>
</tr>
</tbody>
</table>
## Key concepts
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Establish whether a number up to 100 is prime and recall prime numbers up to 19.
- Recognise and use square numbers and cube numbers, and the notation for squared ($^2$) and cubed ($^3$).

### Possible learning intentions
- Identify multiples of numbers
- Explore factors of numbers
- Investigate prime numbers
- Work with square and cube numbers

**Bring on the Maths**: Moving on up!

### Possible success criteria
- Know the meaning of ‘multiple’
- Identify multiples of a given number
- Know the meaning of ‘factor’
- Know how to find factors of a given number
- Know the meaning of ‘common factor’
- Know the meaning of ‘prime number’
- Recall the prime numbers less than 20
- Know how to test if a number up to 100 is prime
- Understand the use of notation for squared and cubed
- Work out the first 10 square numbers
- Work out the first 5 cube numbers

### Prerequisites
- Recall multiplication facts to $12 \times 12$ and associated division facts

**Mathematical language**
- Multiple
- (Common) factor
- Divisible
- Factor pairs
- Prime number, Composite number
- Square number, Cube number
- Power

**Notation**

- $S^2$ is read as ‘$S$ to the power of 2’ or ‘$S$ squared’ and means ‘$2$ lots of $S$ multiplied together’
- $S^3$ is read as ‘$S$ to the power of 3’ or ‘$S$ cubed’ and means ‘$3$ lots of $S$ multiplied together’

**Pedagogical notes**

- ‘Squared’ and ‘cubed’ are special cases of powers. The language ‘to the power of’ can also be introduced to prepare pupils for the future when they will deal with higher powers.
- NCETM: **Glossary**

#### Common approaches

- The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.
- Every classroom has a set of number classification posters on the wall.

### Reasoning opportunities and probing questions
- Kenny says ‘16 is a square number because $8^2 = 16$’. Explain why Kenny is wrong.
- Convince me that 17 is a prime number.
- Show me an example of a multiple of 4. And another. Now find a multiple of 4 that you think no one else in the room will choose.

**NCETM**: **Multiplication and Division Reasoning**

- KM: **Dominoes**. Use the scoring system.
- KM: Use Eratosthenes' sieve to identify prime numbers up to 100.
- KM: **Exploring primes activities**: Numbers of factors.
- KM: **Square numbers**
- NRICH: **Factors and multiples KS2**
- NRICH: Two primes make one square
- NRICH: **Up and down staircases**

**Suggested activities**

- Learning review: [www.diagnosticquestions.com](http://www.diagnosticquestions.com)

**Possible misconceptions**

- Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as ‘a number which is divisible by itself and 1’.
- A common misconception is to believe that $6^2 = 6 \times 2 = 12$. 
### Key concepts
- read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit
- read Roman numerals to 1000 (M) and recognise years written in Roman numerals
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero

### Possible learning intentions
- Work with numbers up to one million
- Understand and use Roman numerals
- Understand and use negative numbers

### Possible success criteria
- Understand place value in numbers with up to seven digits
- Order numbers up to and including those with seven digits
- Write numbers up to and including those with seven digits
- Read numbers up to and including those with seven digits
- Know the meaning of the Roman numerals D and M
- Interpret a year when written in Roman numerals
- Count backwards in whole number steps when negative numbers are included
- Count forwards in whole number steps when negative numbers are included
- Understand and use temperatures below 0°C
- Interpret negative numbers in other contexts

### Prerequisites
- Understand and use place value in four-digit numbers
- Know Roman numerals from I to C
- Read numbers written in Roman numerals up to 100
- Count forwards and backwards in whole number steps

### Mathematical language
- Place value
- Digit
- Roman numerals
- Negative number

#### Notation
See notes about Roman numerals

### Pedagogical notes
- Zero is neither positive nor negative.
- Ensure that pupils read information carefully and check whether the required order is smallest first or greatest first.
- Ensure that pupils can deal with large numbers that include zeros in the HTh and/or H column (e.g. 1 029 628).
- In general it is incorrect to repeat a Roman numeral symbol four times (i.e. XXXX).
- Also, the subtractive method should only be used (1) if subtracting powers of ten (i.e. I, X or C), and (2) if subtracting from the next two higher symbols (for example, I can be subtracted from V or X, but not L, C, D or M). Therefore 49 cannot be written as XXXXIX, or as IL, and must be written as XLIX.

### Reasoning opportunities and probing questions
- Look at this number (1 029 628). Show me another number (with 4, 5, 6, 7 digits) that includes a 9 with the same value. And another. And another ...
- Jenny reads the number 1 029 008 as ‘one million, twenty nine thousand and eight’. Kenny reads the same number as ‘one million, two hundred and nine thousand and eight’. Who is correct? How do you know?
- Convince me that 2014 is MMXIV in Roman numerals
- Convince me that -17°C is colder than -14°C

### Suggested activities
- KM: Roman numeral converter. Note that we use Arabic numerals today! Choose a number and convert it instantly. Can pupils work out the system for numbers above 100?
- KM: Roman numeral times table jigsaw: Use the larger version to start looking at numbers above 100.
- NRICH: Sea level
- NRICH: Tug Harder!
- Learning review

### Possible misconceptions
- Some pupils can confuse the language of large (and small) numbers since the prefix ‘milli-’ means ‘one thousandth’ (meaning that there are 1000 millimetres in a metre for example) while one million is actually a thousand thousand.
- The use of IIII on a clock face suggests that a Roman numeral can be repeated four times, but this is a special case. In general, three is the maximum number of repeats and the subtractive method should be used instead (i.e. IV).
Calculating: addition and subtraction

### Key concepts
- add and subtract numbers mentally with increasingly large numbers
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

### The Big Picture: Calculation progression map

### Possible learning intentions
- Develop mental addition and subtraction skills
- Extend written methods of addition and subtraction
- Solve problems involving addition and subtraction

### Possible success criteria
- Add four-digit numbers and ones, tens, hundreds or thousands mentally
- Subtract four-digit numbers and ones, tens, hundreds or thousands mentally
- Add a three-digit number to a two-digit number mentally (when no bridging of hundreds is required)
- Use column addition for numbers with more than four digits
- Use column subtraction for numbers with more than four digits
- Identify when addition or subtraction is needed as part of solving multi-step problems
- Explain why addition or subtraction is needed at any point when solving multi-step problems
- Solve multi-step problems involving addition and/or subtraction

### Prerequisites
- Add and subtract numbers mentally, including a three-digit number and ones, tens or hundreds
- Use column addition and subtraction for numbers up to four digits
- Estimate the answer to a calculation

### Mathematical language
| Add and subtract numbers mentally, including a three-digit number and ones, tens or hundreds |
| Use column addition and subtraction for numbers up to four digits |
| Estimate the answer to a calculation |

### Pedagogical notes
- Ensure that pupils can deal with column subtractions that include a 0 within the first number; e.g. 48027 – 8437.
- Later in this stage there is a further opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.
- KM: Progression: Addition and Subtraction and Calculation overview
- NCETM: The Bar Model, Subtraction
- NCETM: Glossary

### Common approaches
- To avoid confusion with language, all teachers use ‘sum’ to refer only to the result of an addition. Teachers say ‘complete these calculations’ instead of ‘complete these sums’
- All pupils use books / paper with 7mm squares and ensure that each digit is written in one square
- When carrying, those numbers being carried are placed beneath the answer line
- During column subtraction the language of ‘exchanging’ is used instead of ‘borrowing’
- When exchanging, those numbers being altered or moved are written above the calculation

### Reasoning opportunities and probing questions
- Provide examples of column addition and subtraction with hidden digits. Challenge pupils to find these digits and explain their reasoning.
- Show me an example of a column addition (that includes carrying) with the answer 54192
- Convince me that 56095 – 23622 = 32473

### Suggested activities
- KM: Palindromic numbers
- KM: Pairs in squares
- KM: Interactive target boards
- KM: Maths to Infinity: Addition and subtraction foundations
- NRICH: Journeys in Numberland
- NRICH: Twenty Divided Into Six
- NRICH: Two and Two
- KM: Following on from ‘Two and Two’ above, why is FIVE + TWO = SEVEN impossible? How about THREE + NINE = TWELVE and FORTY + FORTY = EIGHTY? Consider column methods.
- Learning review
- www.diagnosticquestions.com

### Possible misconceptions
- When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166.
- Some pupils incorrectly assume and use commutativity within column subtraction; for example:
  - 7 4 1 2 6
  - 2 3 7 3 4
  - 5 1 6 1 2
- Some pupils may not use place value settings correctly (especially when the numbers have a different number of digits)
### Calculating: multiplication and division

**Key concepts**
- Multiply and divide numbers mentally drawing upon known facts
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

**The Big Picture**
- Calculation progression map

<table>
<thead>
<tr>
<th>Possible learning intentions</th>
<th>Possible success criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Develop mental arithmetic skills</td>
<td>- Use knowledge of multiplication tables when multiplying and dividing mentally</td>
</tr>
<tr>
<td>- Explore multiplication and division of decimals</td>
<td>- Multiply (or divide) a whole number or decimal by 10, 100 or 1000</td>
</tr>
<tr>
<td>- Develop written methods of multiplication</td>
<td>- Know how to set up a long multiplication problem</td>
</tr>
<tr>
<td>- Develop written methods of division</td>
<td>- Use long multiplication to multiply a two- (or three-, or four-) digit number by a two-digit number</td>
</tr>
<tr>
<td>- Solve problems involving multiplication and division</td>
<td>- Understand the method of short division</td>
</tr>
</tbody>
</table>

**Prerequisites**
- Recall multiplication facts for multiplication tables up to 12 × 12
- Recall division facts for multiplication tables up to 12 × 12
- Find factor pairs of a given number
- Understand the commutativity of multiplication
- Multiply and divide a two-digit number by 10, 100
- Multiply a three-digit number by a one-digit number using short multiplication

**Mathematical language**
- Multiply, Multiplication, Times, Product
- Commutative
- Divide, Division
- Divisible
- Divisor, Dividend, Quotient, Remainder
- Factor
- Short multiplication
- Long multiplication
- Short division
- Operation
- Estimate

**Possible misconceptions**
- The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic expressions. Later in this stage there is a further opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.
- KM: Progression: Multiplication and Division and Calculation overview
- NCETM: The Bar Model, Multiplication, Division, Multiplicative reasoning
- NCETM: Glossary

**Common approaches**
All classrooms display a times table poster with a twist
To avoid confusion with language, all teachers use ‘sum’ to refer only to the result of an addition. Teachers say ‘complete these calculations’ instead of ‘complete these sums’
The use of long multiplication is promoted as the ‘most efficient method’.
Short division is promoted as the ‘most efficient method’.
When dealing with remainders in division problems, use the notation ‘r’

**Pedagogical notes**
- The Big Picture: Calculation overview

**Possible success criteria**
- Use knowledge of multiplication tables when multiplying and dividing mentally
- Multiply (or divide) a whole number or decimal by 10, 100 or 1000
- Know how to set up a long multiplication problem
- Use long multiplication to multiply a two- (or three-, or four-) digit number by a two-digit number
- Understand the method of short division
- Use short division to divide a two- (or three-, or four-) digit number by a one-digit number
- Interpret a remainder when carrying out division
- Identify the correct operation(s) required in order to solve a problem
- Identify when knowledge of factors (multiples, squares, cubes) can be used to help solve a problem

**Suggested activities**
- KM: Happy and sad
- KM: Long multiplication template
- KM: Maximise, minimise. Game 2.
- KM: Tens and hundreds. Use Powers of ten to demonstrate connections.
- KM: Maths to Infinity: Multiplying and dividing
- KM: Interactive target boards
- KM: Maths to Infinity: Multiplication and division foundations
- NRICH: Curious Number
- NRICH: Make 100
- NRICH: Dicey Operations. Games 4 and 5.
- Learning review

**Reasoning opportunities and probing questions**
- Find missing digits in otherwise completed long multiplication / short division calculations
- Convince me that 247 × 12 = 2964
- What is the same and what is different: 1344 × 6 and 504 × 16?
- What is wrong with this short division? How can you correct it?

<table>
<thead>
<tr>
<th>0</th>
<th>10</th>
<th>7</th>
<th>r 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>3</td>
<td>6</td>
<td>91</td>
</tr>
</tbody>
</table>

NCETM: Multiplication and Division Reasoning

**Possible misconceptions**
- Some pupils may write statements such as 2 × 8 = 4
- When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example:

```
  0 10 7 r 5
------------------------
8 | 3 6 91
```

3 ÷ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been ‘moved across’ and therefore everything that follows has been correctly carried out based on an early misunderstanding.
### Investigating properties of shapes

#### Key concepts
- use the properties of rectangles to deduce related facts and find missing lengths and angles
- distinguish between regular and irregular polygons based on reasoning about equal sides and angles

#### Possible learning intentions
- Explore the properties of rectangles
- Investigate polygons

#### Possible success criteria
- Know the definition and properties of a rectangle
- Use the properties of rectangles to find missing lengths and angles
- Use the properties of rectangles to find points on a coordinate grid
- Know the definition of a polygon
- Know the difference between a regular and an irregular polygon
- Identify whether or not a polygon is regular
- Use the properties of polygons to find missing lengths and angles

#### Prerequisites
- Identify right angles
- Use coordinates in the first quadrant

**Bring on the Maths**: Moving on up!

**Position and direction**: #2

**Mathematical language**

<table>
<thead>
<tr>
<th>Rectangle</th>
<th>Square</th>
<th>Quadrilateral</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Regular / irregular) polygon, pentagon, hexagon, octagon</td>
<td>(Right) angle</td>
<td>Parallel</td>
</tr>
<tr>
<td>Perpendicular</td>
<td>Coordinates</td>
<td></td>
</tr>
</tbody>
</table>

**Notation**

- Dash notation to represent equal lengths in shapes and geometric diagrams
- Right angle notation (Cartesian) coordinates

**Pedagogical notes**

- Note that a square is a rectangle but a rectangle is not necessarily a square.
- A square is a regular quadrilateral.
- Pupils may also know names of other polygons such as heptagon (7 sides), nonagon (9 sides), decagon (10 sides) and dodecagon (12 sides).

**NCETM**: Glossary

**Common approaches**

- Every classroom has a set of triangle posters and quadrilateral posters on the wall

#### Reasoning opportunities and probing questions

- Convince me that a square is a rectangle
- Show me an example of a hexagon. And another, and another, ...
- What is the same and what is different: [shapes]

**NCETM**: Geometry - Properties of Shapes Reasoning

#### Suggested activities

<table>
<thead>
<tr>
<th>KM: Shape work</th>
<th>Rectangle, Packing squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRICH: Egyptian Rope</td>
<td></td>
</tr>
<tr>
<td>NRICH: Use the virtual geoboard to explore how regular polygons can be made using equally spaced points around a circle, and ways of constructing rectangles on any of the three type of board</td>
<td></td>
</tr>
<tr>
<td>KM: 6 point circles, 8 point circles and 12 point circles can be used to support the above idea</td>
<td></td>
</tr>
</tbody>
</table>

**Learning review**

- [www.diagnosticquestions.com](http://www.diagnosticquestions.com)

#### Possible misconceptions

- Some pupils may think that a ‘regular’ polygon is a ‘normal’ polygon
- Some pupils may think that all polygons have to be regular
- Some pupils may use coordinates the wrong way round; for example, interpreting the point (3,2) as 3 up and 2 across (to the right)
### Visualising and constructing

#### Key concepts
- Identify 3-D shapes, including cubes and other cuboids, from 2-D representations

#### The Big Picture: Properties of Shape progression map

#### Possible learning intentions
- Investigate 3D shapes

#### Possible success criteria
- Identify 3D shapes from photographs
- Identify 3D shapes from sketches
- Identify 3D shapes from nets
- Identify 3D shapes from diagrams on isometric paper
- Construct diagrams of 3D shapes on isometric paper

#### Prerequisites

<table>
<thead>
<tr>
<th>Mathematical language</th>
<th>Pedagogical notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cube</td>
<td>A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid. A cube is a special case of a cuboid, and a cuboid is a special case of a prism. Many pupils struggle to sketch 3D shapes. A good strategy for any type of prism is to draw the cross-section (using squares for guidance), and then draw a second identical shape offset from the first. The matching corners can then be joined with straight lines. Some dotted lines (or rubbing out of lines) will be required. NCETM: <a href="#">Glossary</a>.</td>
</tr>
<tr>
<td>Cuboid</td>
<td></td>
</tr>
<tr>
<td>Cylinder</td>
<td></td>
</tr>
<tr>
<td>Pyramid</td>
<td></td>
</tr>
<tr>
<td>Prism</td>
<td></td>
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<tr>
<td>Cone</td>
<td></td>
</tr>
<tr>
<td>Sphere</td>
<td></td>
</tr>
<tr>
<td>2D</td>
<td></td>
</tr>
<tr>
<td>3D</td>
<td></td>
</tr>
<tr>
<td>Net</td>
<td></td>
</tr>
<tr>
<td>Sketch</td>
<td></td>
</tr>
<tr>
<td>Isometric paper</td>
<td></td>
</tr>
</tbody>
</table>

#### Prerequisites

- Know the names of common 3D shapes

#### Mathematical language

- Cube
- Cuboid
- Cylinder
- Pyramid
- Prism
- Cone
- Sphere
- 2D
- 3D
- Net
- Sketch
- Isometric paper

#### Possible misconceptions

- Pupils must have isometric paper in portrait orientation for it to work correctly.
- When drawing a cube on isometric paper, some students may think they need to join dots to make a square first, and will draw horizontal and vertical lines to attempt to achieve this.
- Correct use of isometric paper must not indicate ‘hidden’ lines.

#### Reasoning opportunities and probing questions

- (Showing photograph / sketch / isometric drawing / net), convince me that this shape is a cuboid / cube / prism / …
- Show me a way to draw a cube. And another. And another …
- Show me a way to draw a 2cm by 3cm by 4cm cuboid on isometric paper. And another. And another …
- What is wrong with this sketch of a cuboid? How can it be changed?

#### Suggested activities

- KM: [Shape work](#): Dice, Opposite numbers, Cutting cubes, Painted cube
- NRICH: [The Third Dimension](#)
- NRICH: [A Puzzling Cube](#)
- NRICH: [Rolling That Cube](#)
- Learning review

- [www.diagnosticquestions.com](http://www.diagnosticquestions.com)

#### NCETM: [Geometry - Properties of Shapes Reasoning](#)

#### Return to overview
## Exploring time

### The Big Picture: Measurement and mensuration progression map

### Key concepts
- solve problems involving converting between units of time
- complete, read and interpret information in tables, including timetables

### Possible learning intentions
- Solve problems involving time
- Interpret information in tables
- Interpret information in timetables

### Possible success criteria
- Identify when it is necessary to convert between units of time to solve a problem
- Choose a correct conversion to use
- Convert a given time into a different unit of time
- Solve a problem involving converting between different units of time
- Identify the types of information arranged in a table
- Read information given in a table
- Interpret the meaning of information given in a table
- Interpret the meaning of information given in a timetable
- Complete a table from given information
- Solve problems that involve interpreting timetables

### Possible misconceptions
- Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point
- Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems
- Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o’clock

### Measures: #1, #2

#### Bring on the Maths: Moving on up!

### Prerequisites
- Read, write and convert time between analogue and digital 12- and 24-hour clocks
- Know how to convert from hours to minutes; minutes to seconds; years to months; weeks to days

### Mathematical language
- Millennium
- Century
- Decade
- Year
- Month
- Week
- Day
- Hour
- Minute
- Second
- Timetable

#### Notation
- 12- and 24-hour clock notation
- 24-hour clock notation can be with or without a colon separating hours and minutes
- Analogue clocks with Arabic or Roman numerals

### Pedagogical notes
- NCETM: Glossary
- Common approaches
  - All pupils solve problems involving the use of local bus and train timetables

### Reasoning opportunities and probing questions
- (Using a timetable) I want to arrive in Chichester by 10:15. Show me a train that I could catch from Portsmouth. And another. What is the latest train I could catch? What time does this train leave Portsmouth?
- Convince me that that are 135 minutes between 1115 and 1:30 p.m.
- Jenny and Kenny are solving a problem that involves planning a journey. They are leaving Chester at 08:12. The journey takes 1 hour and 50 minutes. Jenny thinks that they will arrive at 09:62. Kenny thinks that they will arrive at 10:02. Who do you agree with? Explain your answer.

### Suggested activities
- KM: Timetable progression
- NRICH: Watch the clock
- NRICH: Two clocks
- NRICH: Train timetable
- NRICH: Slow coach

### Learning review
- www.diagnosticquestions.com

### NCETM: Measurement Reasoning
Exploring fractions, decimals and percentages

Key concepts
- compare and order fractions whose denominators are all multiples of the same number
- identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- read and write decimal numbers as fractions [for example, 0.71 = \( \frac{71}{100} \)]
- read, write, order and compare numbers with up to three decimal places
- recognise the per cent symbol (%) and understand that per cent relates to ‘number of parts per hundred’, and write percentages as a fraction with denominator 100, and as a decimal

The Big Picture: Fractions, decimals and percentages progression map

Possible learning intentions
- Explore the equivalence between fractions
- Explore the equivalence between fractions and decimals
- Understand the meaning of percentages

Possible success criteria
- Use diagrams to compare the size of fractions
- Identify when two fractions can be compared easily without a diagram
- Compare two fractions without using a diagram
- Order a set of fractions when the denominators are equal
- Know that fractions can be equivalent
- Know how use a diagram to find a fraction that is equivalent to another fraction
- Know that thousandths is the name of the column beyond hundredths
- Understand that thousandths can be written as fractions or as decimals
- Write a number (less than 1) with one decimal place as a fraction
- Write a number (less than 1) with two decimal places as a fraction
- Read a number with three decimal places
- Compare a set of numbers written to three decimal places
- Compare a set of numbers with a mixed number of decimal places
- Understand that per cent relates to number of parts per hundred
- Understand that a percentage can be written as a fraction with a denominator of 100
- Write any percentage as a decimal

Prerequisites
- Understand the concept of equivalent fractions
- Understand that tenths and hundredths can be written as fractions or as decimals
- Know that \( \frac{1}{4} = 0.25 \), \( \frac{1}{2} = 0.5 \) and \( \frac{3}{4} = 0.75 \)

Mathematical language
- Fraction
- Numerator
- Denominator
- Improper fraction, Proper fraction, Vulgar fraction, Top-heavy fraction
- Tenth, hundredth, thousandth
- Per cent, Percentage
- Decimal
- Equivalent

Notation
- Diagonal fraction bar / horizontal fraction bar

Pedagogical notes
- NRICH: Teaching fractions with understanding
- NCETM: Teaching fractions
- NCETM: Glossary

Common approaches
- All pupils are made aware that ‘per cent’ is derived from Latin and means ‘out of one hundred’
- Teachers use the horizontal fraction bar notation at all times

Possible misconceptions
- Some pupils may read 0.234 as ‘nought point two hundred and thirty four’. This leads to the common misconception that, for example, 0.400 is a number larger than 0.76
- Pupils may not make the connection that a percentage is a different way of describing a proportion
- Some pupils may think that equivalent fractions are found using an additive relationship rather than a multiplicative one: for example, that the fraction 4/5 is equivalent to 6/8

Reasoning opportunities and probing questions
- Show me a fraction that is equivalent to 7/10. And another ...
- Convince me that 6/8 is greater than 7/16
- Jenny says that 0.127 is ‘one hundred and twenty seven thousandths’. Kenny says that 0.127 is ‘one tenth, two hundredths and seven thousandths’. Who do you agree with? Explain your reasoning.
- NCETM: Fractions Reasoning

Suggested activities
- KM: Decimal ordering cards 1
- KM: Fraction action
- KM: Carpets
- NRICH: Spiralling decimals
- NCETM: Activity D - Metre sticks and metre strips
- NCETM: Activity F - Using blank hundred squares
- Learning review
- www.diagnosticquestions.com

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- Learning review
- www.diagnosticquestions.com
<table>
<thead>
<tr>
<th>Pattern sniffing</th>
<th>4 hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Big Picture: Algebra progression map</td>
<td></td>
</tr>
<tr>
<td><strong>Key concepts</strong></td>
<td></td>
</tr>
<tr>
<td>• count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Possible learning intentions</th>
<th>Possible success criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Develop ways of counting</td>
<td>• Count forwards in tens (hundreds, thousands) from any positive number up to 10 000 (100 000, 1 000 000)</td>
</tr>
<tr>
<td></td>
<td>• Count backwards in tens (hundreds, thousands) from any positive number up to 10 000 (100 000, 1 000 000)</td>
</tr>
<tr>
<td></td>
<td>• Count forwards through zero</td>
</tr>
<tr>
<td></td>
<td>• Count backwards through zero</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Prerequisites</th>
<th>Mathematical language</th>
<th>Pedagogical notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Understand place value in numbers with up to seven digits</td>
<td>Forwards Backwards Ascending Descending Pattern Sequence</td>
<td>Pupils have counted forwards and backwards in previous years and units, but this is the first time that 'Pattern Sniffing' appears as a unit in its own right. NCETM: Glossary</td>
</tr>
<tr>
<td>• Read and write numbers up to and including those with seven digits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Count backwards in whole number steps when negative numbers are included</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Count forwards in whole number steps when negative numbers are included</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Common approaches Teachers and pupils refer to numbers less than zero as 'negative' numbers and not 'minus' numbers</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reasoning opportunities and probing questions</th>
<th>Suggested activities</th>
<th>Possible misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Show me a number that is easy (difficult) to count forward in tens (hundreds, thousands). And another. And another ...</td>
<td>NRICH: Tug Harder! NCETM: Activity set B Learning review <a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a></td>
<td>• Some pupils may think the negative number line is:</td>
</tr>
<tr>
<td>• Kenny is counting forwards ... '4060, 4070, 4080, 4090, 5000.' Do you agree with Kenny? Explain your answer.</td>
<td></td>
<td>• Some pupils may bridge straight to the next thousand rather than the next hundred, such as '4060, 4070, 4080, 4090, 5000.'</td>
</tr>
<tr>
<td>• Convince me that one less than -2 is -3 and not -1</td>
<td></td>
<td>• Some pupils may think that 1 million is one more than 9999.</td>
</tr>
</tbody>
</table>
### Measuring Space

#### The Big Picture: Measurement and mensuration progression map
- **8 hours**

#### Key Concepts
- Convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)
- Understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints
- Use all four operations to solve problems involving measure (for example, length, mass, volume, money) using decimal notation, including scaling

#### Possible Learning Intentions
- Convert between measures
- Know and work with common Imperial units
- Solve problems involving measurement
- Solve problems involving money

#### Possible Success Criteria
- Convert between centimetres and metres
- Convert between kilograms and grams
- Use decimal notation when converting between metric units of length, mass and volume/capacity
- Know that one inch is roughly equivalent to 2.5 cm
- Know that one foot is roughly equivalent to 30 cm
- Know that one kilogram is roughly equivalent to 2.2 lb
- Know that one pint is roughly equivalent to 550 ml
- Use rough equivalents between metric and imperial units when solving problems
- Choose the correct operations when solving problems involving measures
- Solving scaling problems involving measure

#### Prerequisites
- Convert between kilometres and metres, centimetres and millimetres
- Convert between litres and millilitres
- Convert between hours and minutes, minutes and seconds
- Use decimal notation to two decimal places when converting between measures

#### Mathematical Language
- Length, distance
- Mass, weight
- Volume
- Capacity
- Metre, centimetre, millimetre
- Kilogram, gram
- Litre, millilitre
- Hour, minute, second
- Inch, foot, yard
- Pound, ounce
- Pint, gallon

#### Notation
- Abbreviations of units in the metric system: m, cm, mm, kg, g, l, ml
- Abbreviations of units in the Imperial system: lb, oz

#### Pedagogical Notes
- Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement.
- The prefix ‘centi-’ means one hundredth, and the prefix ‘milli-‘ means one thousandth. These words are of Latin origin. The prefix ‘kilo-’ means one thousand. This is Greek in origin.

#### Common Approaches
- Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)

#### Reasoning Opportunities and Probing Questions
- Kenny thinks 1.5m = 105cm. Do you agree with Kenny? Explain your answer
- Show me an imperial (metric) unit of measure. And another. And another.
- Convince me that 3.07kg = 3070g.
- Which of the following is the best value for money?
  - 1 litre for £2 or 2 pints for £2
  - 5kg for 40p or 4lbs for 40p
  - 10cm for £2 or 5 inches for £2

#### Suggested Activities
- NRICH: Olympic Starters
- NCETM: Activity D - Converting between metric units
- NCETM: Activity E- Converting between metric and imperial
- Learning review: www.diagnosticquestions.com

#### Possible Misconceptions
- Some pupils may apply incorrect beliefs about place value, such as 2.3 × 10 = 2.30.
- Many conversions within the metric system rely on multiplying and dividing by 1000. The use of centimetres as an ‘extra unit’ within the system breaks this pattern. Consequently there is a frequent need to multiply and divide by 10 or 100, and this can cause confusion about the connections that need to be applied.
- Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point.

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### Investigating angles

#### Key concepts
- Know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles
- Draw given angles, and measure them in degrees (°)
- Identify: angles at a point and one whole turn (total 360°); angles at a point on a straight line and 1/2 a turn (total 180°); other multiples of 90°

#### The Big Picture: Position and direction progression map

#### Possible learning intentions
- Develop knowledge of angles
- Measure angles
- Draw angles

#### Possible success criteria
- Know that angles are measured in degrees
- Know that angles in a full turn total 360°
- Know that angles on a straight line total 180°
- Know that a reflex angle is greater than 180°
- Identify angles at a point
- Identify angles at a point on a straight line
- Estimate the size of angles
- Use a protractor to measure angles less than 180°
- Use a protractor to measure angles greater than 180°
- Use a protractor to draw angles less than 180°
- Use a protractor to draw angles greater than 180°

#### Prerequisites
- Understand that an acute angle is less than a right angle
- Understand that an obtuse angle is greater than a right angle and less than two right angles
- Identify acute angles
- Identify obtuse angles
- Identify acute, obtuse and right angles in shapes
- Compare angles up to two right angles in size
- Order angles up to two right angles in size

#### Mathematical language
- Turn
- Angle
- Degrees
- Right angle
- Acute angle
- Obtuse angle
- Reflex angle
- Protractor

#### Notation
- Right angle notation
- Arc notation for all other angles
- The degree symbol (°)

#### Prerequisites
- The use of degrees as a unit for measuring angles is first introduced in this unit.
- The exact reason for there being 360 degrees in a full turn is unknown.
- There are various theories including it being an approximation of the 365 days in a year and resultant apparent movement of the sun, and the fact that it has so many factors.
- The SI unit for measuring angles in the radian (2π radians in a full turn).
- Napoleon experimented with the decimal degree, or grad (400 grads in a full turn).

#### Pedagogical notes
- NCETM: Glossary

#### Common approaches
- All pupils are taught to use a 180° and a 360° protractor.
- Teachers reference the Babylonian number system for explaining why there are 360° in one whole turn.

#### Reasoning opportunities and probing questions
- Show me an acute (obtuse, reflex) angle. And another. And another.
- Jenny uses a protractor to measure this angle: She writes down 140°. Do you agree with Jenny?
- Convince me how to measure a reflex angle using a 180° protractor.
- Kenny thinks that 90° is an acute angle. Jenny thinks that 90° is an obtuse angle. Who is correct? Explain your answer.

#### Suggested activities
- KM: Angle Vocab
- NRICH: Estimating Angles
- NCETM: Activity A: Logo Challenge 1 – Star Square
- NCETM: Activity C: Equal angles
- NCETM: Activity D: Sorting triangles

#### Learning review
- www.diagnosticquestions.com

#### Possible misconceptions
- Some pupils use the wrong scale on a protractor. For example, they measure an obtuse angle as 60° rather than 120°.
- Some pupils may think that 90° is either an acute or obtuse angle.
- Some pupils may think it is not possible to measure a reflex angle.

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### Prerequisites
- Fractions, decimals & percentages: Bring on the Maths
- Calculating fractions, decimals and percentages

### Possible learning intentions
- Explore mixed numbers
- Calculate with fractions
- Explore fractions, decimals and percentages

#### Bring on the Maths: Moving on up!
Fractions, decimals & percentages: #5

### Mathematical language
- Place value
- Tenth, hundredth, thousandth
- Decimal
- Proper fraction, Improper fraction, top-heavy fraction, vulgar fraction
- Numerator, denominator
- Percent, percentage

#### Notation
- Decimal point
d, h, th notation for tenths, hundredths, thousandths
- Horizontal / diagonal bar for fractions

### Pedagogical notes
- Describe $\frac{1}{3}$ as ‘there are three equal parts and I take one’, and $\frac{1}{4}$ as ‘there are four equal parts and I take three’.
- Be alert to pupils reinforcing misconceptions through language such as ‘the bigger half’.
- To explore the equivalency of fractions make several copies of a diagram with three-quarters shaded. Show that splitting these diagrams with varying numbers of lines does not alter the fraction of the shape that is shaded.
- Later in this stage there is a further opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.
- NCETM: Teaching fractions, Fractions videos, The Bar Model

#### Common approaches
- Teachers use the horizontal fraction bar notation at all times.
- Pupils are encouraged to use the horizontal fraction bar notation at all times.

### Reasoning opportunities and probing questions
- Show me an improper fraction (mixed number). And another.
- Kenny thinks that $\frac{1}{2} + \frac{2}{3} = \frac{3}{12}$. Explain why Kenny is incorrect.
- Jenny thinks that you can only add or subtract fractions if they have the same denominator. Do you agree with Jenny? Explain.
- Show me a fraction, decimal and percentage ‘equivalent family’ (e.g. $\frac{1}{2}$ = 50% = 0.5). And another. And another ...
- Kenny thinks that $\frac{1}{2} \times 5 = \frac{5}{12}$. Do you agree with Kenny? Explain.
- Convince me that $2 \frac{2}{3} \times 3 = 8$ in at least 2 different ways.

### Suggested activities
- KM: The Heinz Matrix 2
- NRICH: Balance of Halves
- NRICH: Route Product
- NRICH: Forgot the Numbers
- NCETM: Activity A - Fractions ITP

#### Learning review
- www.diagnosticquestions.com

### Possible misconceptions
- Some pupils may think that you simply add the numerators and add the denominators when adding fractions.
- Some pupils may think that you simply subtract the numerators and subtract the denominators when subtracting fractions.
- Some pupils may think that you simply multiply both the numerator denominator when multiplying a fraction by a whole number.
- Some pupils may think that you simply multiply the whole number and then the fraction when multiplying a mixed number by a whole number, e.g. $3 \frac{2}{5} \times 2 = 6 \frac{6}{5}$.
### Calculating space

#### Key concepts
- measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
- calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm²) and square metres (m²) and estimate the area of irregular shapes
- estimate volume (for example, using 1 cm³ blocks to build cuboids (including cubes)) and capacity (for example, using water)

#### Possible learning intentions
- Exploring the perimeter of composite shapes
- Calculate areas of rectangles
- Investigate volume and capacity

**Bring on the Maths**: Moving on up!

**Measures**: #4, #5

#### Possible success criteria
- Calculate the perimeter of composite rectilinear shapes when some dimensions are unknown
- Know that the area of a rectangle is given by the formula area = length \( \times \) width
- Know that area can be measured using square centimetres or square metres
- Know the abbreviations cm² and m²
- Calculate the area of a rectangle
- Calculate the area of a square
- Understand how to estimate the area of irregular shapes
- Estimate the area of irregular shapes bounded by straight lines
- Estimate the area of irregular shapes that include curved lines
- Understand the concept of volume
- Understand the concept of capacity
- Estimate volume by using 1 cm³ blocks to build cuboids
- Estimate capacity

#### Prerequisites
- Understand the concept of area
- Understand the concept of perimeter
- Calculate the perimeter of 2D shapes when dimensions are known
- Find the area of rectilinear shapes by counting squares

#### Possible misconceptions
- Some pupils may multiply the numbers to find the perimeter of a shape.
- Some pupils may think that you cannot find the perimeter of a shape unless all the dimensions are given.
- Some pupils may just add the given dimensions, rather than consider any unlabelled dimensions.
- Some pupils may think that you multiply all the numbers to find the area of a rectangle.

#### Mathematical language

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>Dimensions</td>
<td>Square, rectangle</td>
</tr>
<tr>
<td>Composite rectilinear</td>
<td>Polygon</td>
<td>Cube, cuboid</td>
</tr>
<tr>
<td>Millimetre, Centimetre, Metre, Kilometre</td>
<td>Square centimetre, square metre</td>
<td>Cubic centimetre, centimetre cube</td>
</tr>
<tr>
<td>Square unit</td>
<td>Notation</td>
<td>Abbreviations of units in the metric system: km, m, cm, mm, cm², m², cm³</td>
</tr>
</tbody>
</table>

#### Notation

**In terms of perimeter**, this unit focuses solely on the composite rectilinear shapes; i.e., those that are bounded by straight lines that meet at right angles, see NCETM: Y5 Measurement exemplification

**This unit covers three concepts that pupils often confuse. It would be unwise to have a single objective for a lesson (or lessons) covering both area and perimeter for example.**

**NCETM: Glossary**

**Common approaches**

**Pupils are taught to use the 'matching method' (see reasoning section) when estimating area of irregular shapes.**

**When estimating areas of irregular shapes pupils are taught to**

- Pupils cut out and 'feel' the size of one centimetre cube. They make a square metre using metre sticks and have the opportunity to visualise the fact that 10000 cm² are equivalent to 1 m².
- Pupils make and 'feel' the size of one centimetre cube. They have the opportunity to visualise the fact that 1 000 000 cm³ are equivalent to 1 m³.

**Suggested activities**

**KM**: Stick on the Maths SSM7: Area and perimeter

**NRICH**: Area and Perimeter

**NRICH**: Through the Window

**NRICH**: Numerically Equal

**NRICH**: Cubes

**Learning review**

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### Key concepts
- round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000
- round decimals with two decimal places to the nearest whole number and to one decimal place
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

### The Big Picture:
- Number and Place Value progression map

### Possible learning intentions
- Explore ways of approximating numbers
- Explore ways of checking answers

### Possible success criteria
- Approximate any number by rounding to the nearest 10 000
- Approximate any number by rounding to the nearest 100 000
- Approximate any number with two decimal place by rounding to the nearest whole number
- Approximate any number with two decimal place by rounding to one decimal place
- Understand checking as the process of working backwards from the answer to ensure that it makes sense
- Understand estimating as the process of finding a rough value of an answer or calculation
- Estimate addition (subtraction) calculations with up to four digits
- Estimate multiplication calculations that involve multiplying up to four-digit numbers by a single digit
- Estimate multiplication calculations that involve multiplying up to four-digit numbers by a two-digit number
- Estimate division calculations that involve dividing up to a four-digit number by a single digit number

### Prerequisites
- Approximate any number by rounding to the nearest 10, 100 or 1000
- Approximate any number with one decimal place by rounding to the nearest whole number

### Mathematical language
- Approximate (noun and verb)
- Round
- Decimal place
- Check
- Solution
- Answer
- Estimate (noun and verb)
- Accurate
- Accuracy
- Notation
- The approximately equal symbol (=)

### Pedagogical notes
- This unit is an opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.
- Also see big pictures: Calculation progression map and Fractions, decimals and percentages progression map
- NCETM: Glossary

### Common approaches
- All pupils are taught to visualise rounding through the use a number line

### Reasoning opportunities and probing questions
- Convince me that 150 000 rounds to 200 000 to the nearest 100 000
- What is the same and what is different: 1595, 1649, 1534 and 1634
- Benny thinks that 3.16 rounds to 3.1 to one decimal place. Do you agree? Explain your answer.

### Suggested activities
- KM: Stick on the Maths NNS2: Approximating
- KM: Maths to Infinity Rounding
- NCTEM: Activity D
- Learning review: www.diagnosticquestions.com

### Possible misconceptions
- Some pupils may truncate instead of round
- Some pupils may misunderstand the rounding process as one that works from the end of the number; for example 3472 to the nearest 1000 is worked out as 3472 → 3470 → 3500 → 4000.
- Some pupils may round down at the half way point, rather than round up.
### Mathematical movement

**Key concepts**
- Identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed

#### The Big Picture
- Position and direction progression map

<table>
<thead>
<tr>
<th>Possible learning intentions</th>
<th>Possible success criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use transformations to move shapes</td>
<td>Identify a translation</td>
</tr>
<tr>
<td>Bring on the Maths*: Moving on up!</td>
<td>Carry out a translation described using mathematical language</td>
</tr>
<tr>
<td>Position and direction: #1</td>
<td>Identify a reflection</td>
</tr>
<tr>
<td></td>
<td>Carry out a reflection using a mirror line parallel to the axes</td>
</tr>
<tr>
<td></td>
<td>Carry out a reflection using a mirror line parallel to the axes and touching the object</td>
</tr>
<tr>
<td></td>
<td>Carry out a reflection using a mirror line parallel to the axes and crossing the object</td>
</tr>
<tr>
<td></td>
<td>Describe a reflection using mirror lines parallel to the axes</td>
</tr>
<tr>
<td></td>
<td>Know the meaning of ‘congruent’, ‘congruence’, ‘object’, ‘image’</td>
</tr>
<tr>
<td></td>
<td>Understand that a translation produces a congruent image</td>
</tr>
<tr>
<td></td>
<td>Understand that a reflection produces a congruent image</td>
</tr>
</tbody>
</table>

#### Prerequisites
- Use coordinates in the first quadrant
- Describe a translation using mathematical language

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>2-D</td>
</tr>
<tr>
<td>Grid</td>
</tr>
<tr>
<td>Axis, axes, x-axis, y-axis</td>
</tr>
<tr>
<td>Origin</td>
</tr>
<tr>
<td>(First) quadrant</td>
</tr>
<tr>
<td>(Cartesian) coordinates</td>
</tr>
<tr>
<td>Point</td>
</tr>
<tr>
<td>Translation</td>
</tr>
<tr>
<td>Reflection</td>
</tr>
<tr>
<td>Transformation</td>
</tr>
<tr>
<td>Object, Image</td>
</tr>
<tr>
<td>Congruent, congruence</td>
</tr>
</tbody>
</table>

**Pedagogical notes**
- Note that pupils are not yet expected to use an algebraic description of a mirror line (such as $x = 3$).
- The French mathematician Rene Descartes introduced Cartesian coordinates in the 17th century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling.
- Other coordinate systems include grid references, polar coordinates and spherical coordinates.
- There are other types of mathematical movement that pupils will learn about in future stages. The group name for these movements is ‘transformations’. NCETM: Glossary

### Prerequisites
- Use coordinates in the first quadrant
- Describe a translation using mathematical language

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<td>(First) quadrant</td>
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<tr>
<td>Object, Image</td>
</tr>
<tr>
<td>Congruent, congruence</td>
</tr>
</tbody>
</table>

### Learning review
- When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
- When carrying out a reflection some pupils may think that the object and image should be an equal distance from the edge of the grid, rather than an equal distance from the mirror line.
- Some pupils will confuse the order of x-coordinates and y-coordinates.
- When constructing axes, some pupils may not realise the importance of equal divisions on the axes.

<table>
<thead>
<tr>
<th>Possible misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>KM: Moving house</td>
</tr>
<tr>
<td>KM: Stick on the Maths SSM3: Orientation and reflection of shapes</td>
</tr>
<tr>
<td>NRICH: Transformations on a Pegboard</td>
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<tr>
<td>NCETM: Activity A: Translation or Destination</td>
</tr>
</tbody>
</table>

### Reasoning opportunities and probing questions
- (Given a grid with the point (6, 1) indicated) Benny describes this point as (1, 6). Jenny describes the point as (6, 1). Who do you agree with? Why?
- Two vertices of a rectangle are (5, 2) and (4, 0). What could the other two vertices be? How many solutions can you find?
- Always / Sometimes / Never: A mirror line touches the shape that is being reflected
- Always / Sometimes / Never: Translations are easier than reflections

<table>
<thead>
<tr>
<th>Suggested activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>KM: Moving house</td>
</tr>
<tr>
<td>KM: Stick on the Maths SSM3: Orientation and reflection of shapes</td>
</tr>
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<td><a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a></td>
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</tbody>
</table>
### Presentation of data

#### Key concepts
- solve comparison, sum and difference problems using information presented in a line graph

<table>
<thead>
<tr>
<th>Possible learning intentions</th>
<th>Possible success criteria</th>
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</thead>
<tbody>
<tr>
<td>• Solve problems involving graphs</td>
<td>• Understand the difference between a line graph and a bar-line chart</td>
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<td></td>
<td>• Identify when a line graph is an appropriate way to show data</td>
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<td></td>
<td>• Read values from a line graph</td>
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<td></td>
<td>• Answer one-step questions about data in line graphs (e.g. ‘How much?’)</td>
</tr>
<tr>
<td></td>
<td>• Answer two-step questions about data in line graphs (e.g. ‘How much more?’)</td>
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<tr>
<td></td>
<td>• Solve problems using information presented in a line graph</td>
</tr>
</tbody>
</table>

#### Prerequisites
- Interpret and construct a simple bar chart
- Data
- Scale
- Axis
- Graph
- Frequency
- Time graph, Time series
- Line graph
- Bar-line graph, vertical line chart
- Maximum, minimum

<table>
<thead>
<tr>
<th>Mathematical language</th>
<th>Pedagogical notes</th>
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<tbody>
<tr>
<td></td>
<td>William Playfair, a Scottish engineer and economist, introduced the line graph in 1786. Note: Stage 5 focuses on solving problems using information presented in a line graph. Pupils construct simple line (time) graphs in Stage 4. NCETM: <a href="#">Glossary</a></td>
</tr>
</tbody>
</table>

#### Reasoning opportunities and probing questions
- Show me a line graph and tell me a story about it. And another. And another.
- What is the same and what is different: Bar chart, bar-line chart, time graph, line graph?
- Convince me that a line graph is not the same as a bar-line graph.

<table>
<thead>
<tr>
<th>Suggested activities</th>
<th>Possible misconceptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>KM: Stick on the Maths HD4: Frequency diagrams and line graphs</td>
<td>• Some pupils may think that a line graph is appropriate for discrete data</td>
</tr>
<tr>
<td>KM: Stick on the Maths HD7: Line graphs</td>
<td>• Some pupils may think that a line graph is the same as a bar-line chart</td>
</tr>
<tr>
<td>NRICH: Take Your Dog for a Walk</td>
<td>• Some pupils may think that one centimetre represents one unit.</td>
</tr>
<tr>
<td>NCETM: The Mathematics of Mountains</td>
<td></td>
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NCETM: [Statistics Reasoning](#)